Practice Exam 2 Answers — Math 10240

This questions should give an idea of how the exam will be. The actual exam will have fewer questions—probably around 16. Like last time there will be 12 multiple choice and 4 short answer questions.

1. $b^{u-v} = 4$. $b^{v-u} = \frac{1}{4}$. $b^{u/2} = \sqrt{6}$. $b^{-v} = \frac{2}{3}$.
2. omitted.
3. $\lim_{x\to\infty} 2^x = \infty$. $\lim_{x\to-\infty} 2^x = 0$. $\lim_{x\to\infty} 1.001^x = \infty$. $\lim_{x\to 0} 0.95642^x = 1$. Is it true that $\lim_{x\to 1} b^x = b$ for all b? Yes.
4. On first day 1 penny, on second day 2 pennies. On n-th day, 2^{n-1} pennies, so on the 31st day, $2^{31-1} = 2^{30} = 1,073,741,824$ pennies, or \$10,737,418.24. Not a good bet for you.
5. $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$.
6. $\lim_{x\to\infty}(1 + \frac{r}{x})^x = e^r$.
7. $\lim_{x\to\infty}(\frac{x+1}{x})^x = \lim_{x\to\infty}(1 + \frac{1}{x})^x = e$.
8. $t = 66.229$ years.
9. $t = 6.57881$ months. (about 200 days)
10. $\sqrt[89]{20.07} = e^{\frac{1}{89}\ln(\frac{2007}{100})}$. So I guess the button sequence would be $\boxed{2007}$ $\boxed{\div}$ $\boxed{100}$ $\boxed{\ln x}$ $\boxed{\div}$ $\boxed{89}$ $\boxed{e^x}$.
11. $x = 1$.
12. $x = e^{1/e}$.
13. The domain is $(0, \infty)$.
14. $\log_\pi 2007 = \frac{\ln 2007}{\ln \pi}$.
15. No.
16. No.
17. $\log_{1/e} e = -1$.
18. $\log_7 14 - \frac{1}{2}\log_7 4 = 1$.
19. $\log_3 6 + \log_3 12 - \log_3 8 = 2$.
20. $\log_{12} 20 - \log_{12} 240 = -1$.
21. $\ln(e^{\log_9 3^x}) = \frac{x}{2}$.
22. The half-life is about 8.076 days. (The decay rate is $r = -0.0858$).
23. Find $f'(x) = -4x + 1$, so $f'(0) = 1$, so tangent line is given by $y = x - 7$.
24. If $f(t) = t^{17}$ then $\lim_{x\to 0} \frac{(x+9)^{17} - 9^{17}}{x} = f'(9)$. $f'(t) = 17t^{16}$ and $f'(9) = 17(9)^{16} = 31,501,343,210,481,297$. (This number is quite large. Sorry. On the test $17(9)^{16}$ is fine.) This is like the other one: $\lim_{x\to 0} \frac{(x+9)^{17} - 9^{17}}{2x} = \lim_{x\to 0} \frac{1}{2}\frac{(x+9)^{17} - 9^{17}}{x} = \frac{1}{2}f'(9) = \frac{17}{2}9^{16}$.

25. The points are $(-1, \frac{1}{e})$ and $(1, e)$. So the slope of the line connecting them is $\frac{e-\frac{1}{e}}{1-(-1)} = \frac{e-1/e}{2}$.

26. Find $f'(x) = \ln x + 1$, so slope of tangent line at $x = \frac{1}{e}$ is $f'(\frac{1}{e}) = -1 + 1 = 0$. The point is $(\frac{1}{e}, f(\frac{1}{e})) = (\frac{1}{e}, -\frac{1}{e})$, so the line is $y = -\frac{1}{e}$. For the other point the slope of tangent line at $x = e$ is $f'(e) = 1 + 1 = 2$. The point is $(e, f(e)) = (e, e)$ so the equation of the line is $y = 2(x - e) + e$.

27. All tangent lines of a constant function are horizontal, and horizontal lines have slope 0.

28. $f'(a)$ is slope of the tangent line to the point $(a, f(a))$. Or $f'(a)$ is the rate at which the value of $f(x)$ is changing at $x = a$.

29. $\frac{d}{dx}(5f(x)) = 5\frac{d}{dx}f(x) = 5f'(x) = 5(x^7 + e^x)$.

30. Use the product rule: $g'(x) = (x^7 - 1)'f(x) + (x^7 - 1)f'(x)$. Use the chain rule $\frac{d}{dx}f(x^2) = f'(x^2)\frac{d}{dx}x^2 = \frac{2x}{2x^2+5}$. Also use the chain rule $\frac{d}{dx}f(\ln x) = f'(x^2)\frac{d}{dx}\ln x = \frac{1}{x(2(\ln x)+5)}$. (If you wondered which function has this derivative try $f(x) = \frac{1}{2}\ln(2x + 5)$).

31. $\lim_{x\to 2} \sqrt{\frac{x^2-4}{x-2}} = \sqrt{\lim_{x\to 2} \frac{x^2-4}{x-2}} = \sqrt{4} = 2$.

32. This is the derivative of e^t at $t = 0$. Since $\frac{d}{dt}e^t = e^t$, $\lim_{x\to 0} \frac{e^x-1}{x} = e^0 = 1$.

33. $\frac{d}{dx}x^{51} = 51x^{50}$

34. $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.

35. $\frac{d}{dx}\sqrt[3]{x} = \frac{1}{3}x^{-2/3}$.

36. $f'(x) = -\frac{4}{5}x^{-9/5}$.

37. $f'(x) = 2007x^{2006} - 56056x^{1000}$.

38. $\frac{d}{dx}\frac{\ln x}{x} = \frac{1-\ln x}{x^2}$.

39. $\frac{d}{dx}e^{x^2-x} = (2x - 1)e^{x^2-x}$.

40. $\frac{d}{dx}e^x = e^x$.

41. $\frac{d}{dx}\ln\sqrt{x} = \frac{1}{2x}$.

42. $\frac{d}{dx}xe^x = e^x(1 + x)$.

43. $\frac{d}{dx}\ln(2x + 1) = \frac{2}{2x+1}$.

44. Have $s(t) = 23 + 7t + 15t^2 - t^3$. So $v(t) = s'(t) = 7 + 30t - 3t^2$ and $a(t) = v'(t) = 30 - 6^t$. This is negative for $t \geq 5$ so yes.

45. $h(t) = -16t^2 + 20t + 5$ so $v(t) = h'(t) = -32t + 20$. $v(1) = -12$. Velocity is negative when $t \geq \frac{5}{8}$. If the velocity is negative the ball is falling to the ground (as opposed to raising away from the ground).

46. Remember $e^{t^2} = e^{(t^2)}$. Find $g'(t) = 2te^(t^2)$. Slope of tangent line at $t = 1$ is $g'(1) = 2e$.

47. The function is $f(x) = \sqrt{x}$. Find $f'(x) = \frac{1}{2\sqrt{x}}$. The base point for the approximation is $a = 1600$. The slope of f at 1600 is $f'(1600) = \frac{1}{80}$. The equation for the tangent line at $x = 1600$ is $y = \frac{1}{80}(x - 1600) + 40$. Estimate $f(1700) \approx \frac{1}{80}(1700 - 1600) + 40 = 41.25$.

48. $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

49. $\frac{x-3}{x-3}$ is not differentiable at $x = 3$ since it is not continuous there.