

Calculus B Activity

Inverse Functions

January 25, 2007

Name _____

Section _____

This activity is graded out of 10 points. Each of your group members will receive their own copy of the activity. You should work collaboratively to answer all the questions on the activity, and then you will turn in all your papers together, with the sheet you want graded **on top**. For your group to get full credit on this activity, everyone must have evidence of work on their papers (2 points).

1. Let $f(x) = \ln(x^4)$.

(a) Find $f'(x)$.

(b) Let g be the inverse function of f . Find $g(x)$.

(c) What is the domain of $g(x)$?

(d) Use logarithmic differentiation to find $g'(x)$.

(e) Check your answer to part (d) by using the formula $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$.

(f) Show $g'(x) = \frac{1}{f'(g(x))}$.

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2. Consider the inverse functions

$$f(x) = x^2, \quad g(x) = \sqrt{x}, \quad x \geq 0$$

- (a) Graph the functions together on the axes provided.
- (b) Draw tangent lines to the graph of f at $(1, 1)$, $(2, 4)$, and $(3, 9)$.
- (c) Draw tangent lines to the graph of g at $(1, 1)$, $(4, 2)$, and $(9, 3)$.
- (d) What do you observe about the lines and functions you drew ?

- (e) From the above observation, describe the geometric relationship between the tangent line to a function at a point (a, b) and the tangent of its inverse at the point (b, a) .

- (f) How does this relate to the formula for the derivative of an inverse function?

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3. (a) Calculate the following definite integrals:

$$\int_0^1 x^2 dx =$$

$$\int_0^1 \sqrt{y} dy =$$

$$\int_0^1 x^3 dx =$$

$$\int_0^1 y^{\frac{1}{3}} dy =$$

- (b) Suppose that f is a continuous and strictly increasing function on $[0, 1]$ with $f(0) = 0$ and $f(1) = 1$.

If $\int_0^1 f(x) dx = \frac{2}{5}$, calculate $\int_0^1 f^{-1}(y) dy$. Justify your answer.

Hint: Think about your calculations in part (a) and draw a picture.