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## Activity \# 3

1. Your company has just acquired Michiana's finest paper cup manufacturer. The current design can hold $2 \sqrt{3}$ cubic inches of water, but your boss thinks you can do better. The machine makes cups which are conical and uses paper that is 3 inches wide. Can you find a radius and height for a cup that will hold more water?
(a) What is the relationship between the radius $(r)$, height $(h)$, and paper width $(s=3)$ ?

(b) Use an integral to find the volume of a paper cup in terms of $r$ and $h$. What is the volume in terms of $h$ alone?
(c) What value of $h$ gives the largest volume?
(d) Is the maximum volume more than $2 \sqrt{3}\left(=\frac{6}{\sqrt{3}}\right)$ ? What are $r$ and $h$ ?
2. Your boss then comes up with the brilliant idea that the cups be redesigned as in the following diagram. The curve is $\frac{1}{x}$ and the segment from 1 to $a$ is rotated around the x axis.

(a) What is the volume of this cup in terms of $a$ ?
(b) She also wants it to hold 4 cubic inches of water. What value of $a$, if any, achieves this? (Hint: What is the limit of the volume as $a$ goes to infinity?)
