Activity # 3

- 1. Your company has just acquired Michiana's finest paper cup manufacturer. The current design can hold $2\sqrt{3}$ cubic inches of water, but your boss thinks you can do better. The machine makes cups which are conical and uses paper that is 3 inches wide. Can you find a radius and height for a cup that will hold more water?
 - (a) What is the relationship between the radius (r), height (h), and paper width (s = 3)?



(b) Use an integral to find the volume of a paper cup in terms of r and h. What is the volume in terms of h alone?

(c) What value of h gives the largest volume?

(d) Is the maximum volume more than $2\sqrt{3} \ (=\frac{6}{\sqrt{3}})$? What are r and h?

2. Your boss then comes up with the *brilliant* idea that the cups be redesigned as in the following diagram. The curve is $\frac{1}{x}$ and the segment from 1 to a is rotated around the x axis.



(a) What is the volume of this cup in terms of a?

(b) She also wants it to hold 4 cubic inches of water. What value of a, if any, achieves this? (Hint: What is the limit of the volume as a goes to infinity?)