Homework 5: Non-context-free languages

CSE 30151 Spring 2017

Due 2017/03/23 at 11:55pm

Instructions

- Create a PDF file (or files) containing your solutions.
- Please name your PDF file(s) as follows:
  - If you're making a complete submission, please name your PDF file `netid-hw5.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw5-123.pdf`, where `123` is replaced with the problems you are submitting at this time.
- Submit your PDF file in Sakai. Don't forget to click Submit!

Problems

Each problem is worth 10 points.

1. **Pumping lemma for CFLs**. Use the pumping lemma to show that the following languages are not context free:

   (a) [Problem 2.30a] $\{\mathtt{0}^n\mathtt{1}^n\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$

   (b) [Problem 2.31] $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w = w^R$ and w has an equal number of 0s and 1s$\}$

   (c) $\{\mathtt{5m}^{i_1}\mathtt{5m}^{i_2}\cdots\mathtt{5m}^{i_k} \mid k \geq 1, i_1 > i_2 > \cdots > i_k\}$. For example, `5mm5m` is accepted, but `5m5mm` is rejected. (This is arguably the set of all grammatical English numbers involving the words "five" (`5`) and "million" (`m`).)

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ, define the relation $w \mathrel{\mathring{=}} w'$ to be true iff w' is a permutation of

w, that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\begin{aligned} \mathit{SCRAMBLE}(w) &= \{w' \mid w' \mathrel{\mathring{=}} w\} \\ \mathit{SCRAMBLE}(L) &= \bigcup_{w \in L} \mathit{SCRAMBLE}(w). \end{aligned}$$

(a) Show that if $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ and L is a regular language over Σ, then $\mathit{SCRAMBLE}(L)$ is context-free.

(b) Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Show that there is a regular language over Σ such that $\mathit{SCRAMBLE}(L)$ is not context-free.

3. **Closure and non-closure properties**

(a) [Problem 2.18] Let C be any context-free language, and let R be any regular language. Prove that the language $C \cap R$ is context-free. Hint: Product construction.

(b) [Exercise 2.2a] Use the languages

$$\begin{aligned} A &= \{\mathtt{a}^m \mathtt{b}^n \mathtt{c}^n \mid m, n \geq 0\} \\ B &= \{\mathtt{a}^n \mathtt{b}^n \mathtt{c}^m \mid m, n \geq 0\} \end{aligned}$$

to prove that context-free languages are *not* closed under intersection.

(c) [Exercise 2.2b] Use (b) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.