Homework 7: Undecidability

CSE 30151 Spring 2017

Due 2017/04/20 at 11:55pm

Instructions

- Create a PDF file (or files) containing your solutions.
- Please name your PDF file(s) as follows:
  - If you're making a complete submission, please name your PDF file `netid-hw7.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw7-123.pdf`, where `123` is replaced with the problems you are submitting at this time.
- Submit your PDF file in Sakai. Don't forget to click Submit!

Problems

In all of the following problems, any Turing machines that you write can be written as high-level descriptions.

1. **Bounds checking**
   (a) [Problem 5.14] Prove that it is undecidable whether a Turing machine M, on input w, ever attempts to move its head past the left end of the tape.
   (b) Prove that it is decidable whether a Turing machine M, on input w, ever attempts to move its head past the right end of the input string w.

2. ***The Power of 10*** is a set of rules for writing mission-critical code developed at JPL.[1] Among other things, the rules have the following properties:

   - If a TM complies with the rules, then it is a decider, that is, it always halts on every input.[2]
   - It is decidable whether a TM complies with the rules.

   In this problem, we'll show that any such set of rules will be incomplete in the sense that there is a language L_2 that is decidable, yet no TM that decides L_2 complies with the rules.

   Consider the language

   $$L_2 = \{\langle M \rangle \mid M \text{ is a TM that complies with the rules and rejects } \langle M \rangle\}.$$

   (a) Prove that L_2 is decidable.

   (b) Prove that any TM that decides L_2 must not comply with the rules.

3. **A preview of Mire.** In this problem, we'll prove that it is undecidable, given two strings x and y and a finite transducer T, whether x can be transformed into y by applying T zero or more times.

   For example, if T is the transducer defined by the regular transduction expression `(0|1)*(:0)`, which appends a `0` to a string, then we can get from $x =$ `101` to $y =$ `101000` by three applications of T. But we can't get from $x =$ `101` to $y =$ `10101`.

   (a) Given a TM M, show how to construct a finite transducer T (or a regular transduction expression is fine too) that simulates one step of M. More formally, we want T to transform uqv into $u'q'v'$ if and only if M's configuration uqv yields configuration $u'q'v'$ in one step.[3]

   (b) Show how to convert any TM M into an equivalent TM M' that has a single accepting configuration, namely q_{accept}, the configuration with the head at the left end of a blank tape. (This is not difficult, but is needed for the next part.)

   (c) Prove that it is undecidable, given strings x and y and a finite transducer T, whether x can be transformed into y by applying T zero or more times.

[1] `http://bit.ly/powof10`

[2] The rules also allow for a TM that never halts on any input; this is the desired behavior for, e.g., a daemon. But let's ignore this case.

[3] Please see Sipser, pages 168–169, for an explanation of configurations and the *yields* relation.