Homework 5: Non-context-free languages

CSE 30151 Spring 2018

Due 2018/03/22 at 10:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.

- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:

    - If you're making a complete submission, name it *netid*-hw5.pdf, where *netid* is replaced with your NetID.
    - If you're submitting some problems now and want to submit other problems later, name it *netid*-hw5-123.pdf, where 123 is replaced with the problem numbers you are submitting at this time.

- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

Each problem is worth 10 points.

1. **Pumping lemma for CFLs**. Choose **two out of three** of the following languages and use the pumping lemma to show that they are not context free. If you do all three, we'll give credit for the best two.

    (a) [Problem 2.30a] $A = \{\mathtt{0}^n\mathtt{1}^n\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$.

    (b) [Problem 2.31]

    $$B = \{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w = w^R \text{ and } w \text{ has an equal number of } \mathtt{0}\text{s and } \mathtt{1}\text{s}\}.$$

    (c) [cf. HW3, Q2b] The language over $\Sigma = \{\mathtt{0},\mathtt{1},\mathtt{+},\mathtt{=}\}$:

    $$ADD = \{x\mathtt{=}y\mathtt{+}z \mid x,y,z \text{ are binary natural numbers, and } x = y + z\}.$$

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ, define the relation $w \overset{\circ}{=} w'$ to be true iff w' is a permutation of w, that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\text{SCRAMBLE}(w) = \{w' \mid w' \overset{\circ}{=} w\}$$
$$\text{SCRAMBLE}(L) = \bigcup_{w \in L} \text{SCRAMBLE}(w).$$

For example,

$$\text{SCRAMBLE}(0^*1^*) = \{0, 1\}^*$$
$$\text{SCRAMBLE}((01)^*) = \{w \in \{0, 1\}^* \mid w \text{ has an equal number of } 0\text{s and } 1\text{s}\}.$$

(a) Let $\Sigma = \{a, b\}$. Show that for any regular language L over Σ, $\text{SCRAMBLE}(L)$ is context-free.

(b) Let $\Sigma = \{a, b, c\}$. Show that there exists a regular language over Σ such that $\text{SCRAMBLE}(L)$ is *not* context-free.

3. **Non-closure properties**

(a) [Exercise 2.2a] Use the languages

$$A = \{a^m b^n c^n \mid m, n \geq 0\}$$
$$B = \{a^n b^n c^m \mid m, n \geq 0\}$$

to prove that context-free languages are *not* closed under intersection.

(b) [Exercise 2.2b] Use (a) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.