Homework 5: Non-context-free languages

CSE 30151 Spring 2020

Due Friday, 2020/03/27 at 5:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw5.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw4-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

1. **Pumping lemma for CFLs**. For each of the following languages, use the pumping lemma to show that it is not context free.

   (a) [Problem 2.31]

   $$B = \{w \in \{\texttt{0}, \texttt{1}\}^* \mid w = w^R \text{ and } w \text{ has an equal number of } \texttt{0}\text{s and } \texttt{1}\text{s}\}.$$

   (b) [cf. HW3, Q2b] The language over $\Sigma = \{\texttt{0}, \texttt{1}, \texttt{+}, \texttt{=}\}$:

   $$ADD = \{x \texttt{=} y \texttt{+} z \mid x, y, z \text{ are binary natural numbers, and } x = y + z\}.$$

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ, define the relation $w \mathrel{\mathring{=}} w'$ to be true iff w' is a permutation of

w, that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\text{SCRAMBLE}(w) = \{w' \mid w' \mathrel{\mathring{=}} w\}$$
$$\text{SCRAMBLE}(L) = \bigcup_{w \in L} \text{SCRAMBLE}(w).$$

For example,

$$\text{SCRAMBLE}(\texttt{0}^*\texttt{1}^*) = \{\texttt{0}, \texttt{1}\}^*$$
$$\text{SCRAMBLE}((\texttt{01})^*) = \{w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ has an equal number of } \texttt{0}\text{s and } \texttt{1}\text{s}\}.$$

(a) Let $\Sigma = \{\texttt{a}, \texttt{b}\}$. Prove that for any regular language L over alphabet Σ, SCRAMBLE(L) is context-free. (A hint or two will be posted on Piazza.)

(b) Let $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$. Prove that there exists a regular language L over alphabet Σ such that SCRAMBLE(L) is *not* context-free.

3. **Non-closure properties**

(a) [Exercise 2.2a] Use the languages

$$A = \{\texttt{a}^m\texttt{b}^n\texttt{c}^n \mid m, n \geq 0\}$$
$$B = \{\texttt{a}^n\texttt{b}^n\texttt{c}^m \mid m, n \geq 0\}$$

to prove that context-free languages are *not* closed under intersection.

(b) [Exercise 2.2b] Use (a) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.