

# Homework 7: Undecidability

CSE 30151 Spring 2020

Due 2020/04/17 at 5:00pm

## Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it *netid-hw7.pdf*, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it *netid-hw7-123.pdf*, where 123 is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

## Problems

In all of the following problems, any Turing machines that you write can be written as high-level descriptions.

### 1. Bounds checking

- (a) [Problem 5.14; 5.30 intl.] Prove that it is undecidable whether a Turing machine  $M$ , on input  $w$ , ever attempts to move its head past the left end of the tape. Your answer should be a reduction from another undecidable problem (don't use Rice's Theorem).
- (b) Prove that it is decidable whether a Turing machine  $M$ , on input  $w$ , ever attempts to move its head past the right end of the input string  $w$ . Your answer should be a high-level description of a TM.

2. **The Power of 10** is a set of rules for writing mission-critical code developed at JPL.<sup>1</sup> Let us call a Turing machine that complies with these rules *10-compliant*. All that you need to know about 10-compliance is:
- A 10-compliant Turing machine always halts on every input.<sup>2</sup>
  - It is decidable whether a Turing machine is 10-compliant.

In this problem, we'll show that any such set of rules will be incomplete in the sense that there is a language  $L_2$  that is decidable, yet no TM that decides  $L_2$  complies with the rules.

Consider the language

$$L_2 = \{\langle M \rangle \mid M \text{ is a 10-compliant TM that rejects } \langle M \rangle\}.$$

- (a) Prove that  $L_2$  is decidable.
  - (b) Prove that any TM that decides  $L_2$  must not be 10-compliant.
  - (c) Where in your solution to (a) is the violation of 10-compliance?
3. **Rice's Theorem.** Let  $P$  be any property of Turing-recognizable languages that is *nontrivial*, that is, it isn't always true, and it isn't always false.

Rice's theorem [Problem 5.28; 5.16 intl.] says that it is undecidable, given a Turing machine  $M$ , whether  $\mathcal{L}(M)$  has property  $P$ . Once you understand the statement of Rice's theorem, then the following problems should be easy (don't overthink them):

- (a) [Problem 5.30c; 5.18 intl.] Use Rice's Theorem to prove that it is undecidable whether a Turing machine  $M$  recognizes the language  $\Sigma^*$ .
- (b) [Problem 5.29; 5.17 intl.] Show that both conditions in Rice's Theorem are necessary, by:
  - showing that the two trivial properties ( $P$  is always false and  $P$  is always true) are decidable;
  - giving an example of a property of Turing machines – as opposed to the languages they recognize – that is decidable.

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<sup>1</sup><http://bit.ly/powof10>

<sup>2</sup>The rules also allow for a TM that never halts on any input; this is the desired behavior for, e.g., a daemon. But let's ignore this case.