Homework 2: DFAs and NFAs

Theory of Computing (CSE 30151), Spring 2023

Due 2023-02-03 at 11:59pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw2.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw2-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Canvas.

Problems (10 points each)

1. **Designing finite automata** Define, for all $k > 0$,

   $$D_k = \{w \in \{\mathtt{0}, \ldots, \mathtt{9}\}^* \mid w \text{ is the decimal representation of a multiple of } k\},$$

   where ε is considered to represent the number 0. For example, the strings ε, `0`, `88`, and `088` all belong to D_2, but `99` and `099` do not.

   (a) Write a DFA D_2.

   (b) Write a DFA for D_3.

   (c) Prove that D_6 is regular. An explicit DFA is not necessary.

2. **Nondeterminism** Consider the following language:

   $$L_2 = \{uv \mid u, v \in \{\mathtt{a}, \mathtt{b}\}^*, u \text{ contains an even number of } \mathtt{a}\text{'s, and } v \text{ contains an even number of } \mathtt{b}\text{'s}\}$$

Note that as long as there is *some* way of cutting a string into u and v so as to satisfy the constraints, it's in L_2. So $\texttt{ba} \in L_2$, because $u = \texttt{b}$ has an even number (0) of a's and $v = \texttt{a}$ has an even number (0) of b's. But $\texttt{ab} \notin L_2$, because every way of cutting it violates a constraint:

$u = \varepsilon$	$v = \texttt{ab}$	v has odd number of b's
$u = \texttt{ab}$	$v = \varepsilon$	u has odd number of a's
$u = \texttt{a}$	$v = \texttt{b}$	both u and v violate a constraint

(a) Write an NFA N_2 that recognizes L_2.

(b) For $n = 1, \ldots, 4$, show the accepting path (as a sequence of states) for $\texttt{bab}^n$ through N_2, and show where the boundary between u and v occurs.

(c) Convert N_2 to a DFA M_2 using the subset construction (Theorem 1.39).

(d) For $n = 1, \ldots, 4$ show the accepting path for $\texttt{bab}^n$ through M_2. Does M_2 "know" where the boundary between u and v is?

3. **Procrustean closure properties.** Let Σ be an alphabet, and let $L_3 = \{\texttt{theory}, \texttt{of}, \texttt{computing}\}$ be an example language.

(a) For any $w = w_1 w_2 \cdots w_{n-1} w_n$, define

$$\text{STRETCH}(w_1 w_2 \cdots w_n) = w_1 w_1 w_2 w_2 \cdots w_{n-1} w_{n-1} w_n w_n.$$

This induces an operation on languages,

$$\text{STRETCH}(L) = \{\text{STRETCH}(w) \mid w \in L\}.$$

For example,

$$\text{STRETCH}(L_3) = \{\texttt{tthheeoorryy}, \texttt{ooff}, \texttt{ccoommppuuttiinngg}\}.$$

Prove that if L is a regular language, then STRETCH(L) is also regular.

(b) For any $w = w_1 w_2 \cdots w_{n-1} w_n$ with $n \geq 2$, define

$$\text{CHOP}(w_1 w_2 \cdots w_{n-1} w_n) = w_2 \cdots w_{n-1}.$$

This induces an operation on languages,

$$\text{CHOP} = \{\text{CHOP}(w) \mid w \in L \text{ and } |w| \geq 2\}.$$

For example,

$$\text{CHOP}(L_3) = \{\texttt{heor}, \varepsilon, \texttt{omputin}\}.$$

Prove that if L is a regular language, then CHOP(L) is also regular.