# Homework 1: Strings and languages

CSE 30151 Fall 2020

Due: 2020/08/21 5:00pm

## Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.

- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:

  - If you're making a complete submission, name it *netid*-hw1.pdf, where *netid* is replaced with your NetID.

  - If you're submitting some problems now and want to submit other problems later, name it *netid*-hw1-123.pdf, where 123 is replaced with the problem numbers you are submitting at this time.

- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

## Problems (10 points each)

1. **Proof practice.**

   (a) Convert this paragraph proof to a statement-reason proof. Please be sure to write which statement(s) each statement depends on.

   > To show: If $s$ is a string, every substring of a substring of $s$ is a substring of $s$.

   > Proof: Let $y$ be a substring of $s$, that is, $s = xyz$ for some $x$, $z$; and let $v$ be a substring of $y$, that is, $y = uvw$ for some $u$, $w$. Then $s = xuvwz$, so $v$ is a substring of $s$.

   (b) Convert this statement-reason proof to a paragraph proof.

   > To show: If $w$ is a string, every prefix of a suffix of $w$ is a suffix of a prefix of $w$.

1. $v$ is a suffix of $w$     Given
2. $y$ is a prefix of $v$     Given
3. $\exists x$ s.t. $xv = w$     (1), def. suffix
4. $\exists z$ s.t. $yz = v$     (2), def. prefix
5. $xyz = w$     (3), (4), substitution
6. $xy$ is a prefix of $w$     (5), def. prefix
7. $y$ is a suffix of $xy$     (6), def. suffix

2. **String homomorphisms.** If $\Sigma$ and $\Gamma$ are finite alphabets, define a *string homomorphism* to be a function $\phi : \Sigma^* \to \Gamma^*$ that has the property that for any $u, v \in \Sigma^*$, $\phi(uv) = \phi(u)\phi(v)$.

   For example, the function $\phi : \{0, \ldots, 9, A, \ldots, F\}^* \to \{0, 1\}^*$ that converts a hexadecimal number with $n \geq 0$ digits into a binary number with $4n$ bits is a string homomorphism:

   $$\phi(\varepsilon) = \varepsilon$$
   $$\phi(0) = 0000$$
   $$\phi(A) = 1010$$
   $$\phi(CAB) = 110010101011$$

   Intuitively, a string homomorphism does a "search and replace" where each symbol is replaced with a (possibly empty) string. Prove this more formally: that is, prove that if $\phi$ is a string homomorphism, then for any $w = w_1 \cdots w_n$ (where $n \geq 0$ and $w_j \in \Sigma$ for $1 \leq j \leq n$), we have

   $$\phi(w) = \phi(w_1) \cdots \phi(w_n). \tag{*}$$

   Use induction on $n$.

   (a) State and prove the base case ($n = 0$).
   (b) Assume that (*) is true for $n = i$ and prove (*) for $n = i + 1$.

3. **Finite and cofinite.** For any alphabet $\Sigma$, let FINITE be the set of all finite languages over $\Sigma$, and let

   $$\mathsf{coFINITE} = \{L \mid \overline{L} \in \mathsf{FINITE}\}$$

   (where, for any language $L$ over $\Sigma$, $\overline{L}$ is the complement of $L$, that is, $\Sigma^* \setminus L$). For example, $\Sigma^*$ is in coFINITE because its complement is $\emptyset$, which is finite. (Please think carefully about this definition, and note that coFINITE isn't the same thing as $\overline{\mathsf{FINITE}}$.)

   (a) Are there any languages over $\Sigma$ in FINITE∩coFINITE? Prove your answer.
   (b) Are there any languages over $\Sigma$ that are *not* in FINITE∪coFINITE? Prove your answer.