Homework 2: DFAs and NFAs

CSE 30151 Fall 2020

Due 2020/08/28 at 5:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw2.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw2-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems (10 points each)

1. **Designing finite automata** Define, for all $k > 0$,

   $$D_k = \{w \in \{\texttt{0}, \ldots, \texttt{9}\}^* \mid w \text{ is the decimal representation of a multiple of } k\},$$

   where ε is considered to represent the number 0. For example, the strings ε, `0`, `88`, and `088` all belong to D_2.

   (a) Write a DFA for D_2.

   (b) Write a DFA for D_3.

   (c) Prove that for any $k > 0$, D_k is regular, by describing how to write the formal description of a DFA $M = (Q, \{\texttt{0}, \ldots, \texttt{9}\}, \delta, s, F)$ in terms of k. Hint: appending a digit d to a number x is equivalent to doing $x \leftarrow 10x+d$.

2. **Nondeterminism** Consider the following language:

$$L_2 = \{uv \mid u, v \in \{\texttt{a}, \texttt{b}\}^*, \text{ } u \text{ contains an even number of } \texttt{a}\text{'s, and } v \text{ contains an even number of } \texttt{b}\text{'s}\}$$

Note that as long as there is *some* way of cutting a string into u and v so as to satisfy the constraints, it's in L_2. So $\texttt{ba} \in L_2$, because $u = \texttt{b}$ has an even number (0) of a's and $v = \texttt{a}$ has an even number (0) of b's.

(a) Write an NFA N_2 that recognizes L_2.

(b) What is the accepting path for $\texttt{bab}^n$ through N_2? You can show the path for $\texttt{bab}, \texttt{babb}, \texttt{babbb}, \ldots$ until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.

(c) Convert N_2 to a DFA M_2 using the subset construction (Theorem 1.39).

(d) What is the accepting path for $\texttt{bab}^n$ through M_2? Again, you can show the path for $\texttt{bab}, \texttt{babb}, \texttt{babbb}, \ldots$ until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.

3. **Regular/raluger** In the following, we'll use the language L_3 as an example (but the results must be proved for all L):

$$L_3 = \{\texttt{deed}, \texttt{deer}, \texttt{red}, \texttt{redder}, \texttt{reed}\}.$$

(a) Recall that

$$L^R = \{w \mid w^R \in L\}.$$

For example, $L_3^R = \{\texttt{deed}, \texttt{reed}, \texttt{der}, \texttt{redder}, \texttt{deer}\}$. Prove that if L is regular, then L^R is also regular.

(b) Define

$$\text{DOPPELGANGERS}(L) = \{w \mid w \in L \text{ and } w \in L^R\}.$$

For example, $\text{DOPPELGANGERS}(L_3) = \{\texttt{deed}, \texttt{reed}, \texttt{deer}, \texttt{redder}\}$. Prove that if L is regular, then $\text{DOPPELGANGERS}(L)$ is also regular.

(c) **(Optional, not for credit)** Define

$$\text{HALF}(L) = \{w \mid ww^R \in L\}.$$

For example, $\text{HALF}(L_3) = \{\texttt{de}, \texttt{red}\}$. Prove that if L is regular, then $\text{HALF}(L)$ is also regular.