Math 10120 — Spring 2013

Some more practice problems for the final

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1. Of the 75 students in Math 10120, 45 are women. Once during each lecture, I pick a student at random to ask a question to. There are 41 lectures during the semester. **Use the normal approximation to the binomial** to estimate the probability that during the course of the semester, I pick at least 30 men to ask questions to?

   Solution: If X is the number of men I pick, then X is binomial with $n = 41$ and $p = 30/75 = .4$. We have $\mu = E(X) = np = 16.4$ and $\sigma = \sqrt{npq} \approx 3.14$. We want $\Pr(X \geq 30)$. Using the normal approximation, we calculate $\Pr(X \geq 29.5)$ where X is normal with mean 16.4, standard deviation 3.14. Normalizing to a standard normal, this is the same as $\Pr(Z \geq 4.18)$. This is (basically) 0.

2. When I throw a discus, my average distance is 50 meters, with standard deviation 5. My throw distances are definitely **not** normally distributed, but still, from the information given, you can say things for certain. For example: With 90% probability, my throws will always be between $50 - a$ meters and $50 + a$ meters. What is a?

   Solution: Using Tchebychev, the probability of a throw being in the range $50 - a$ meters to $50 + a$ meters is at least $1 - (\sigma^2/a^2) = 1 - (25/a^2)$. To make this .9, we need $25/a^2 = .1$, or $a^2 = 250$, or $a \approx 15.8$.

3. I want to determine something about the speeds at which cars drive past the pedestrian crossing at the corner of Vaness and Twyckenham. I take measurements of 10 cars, and get the following readings from my speed gun (all measurements in miles per hour):

   $$25, \quad 35, \quad 45, \quad 50, \quad 55, \quad 35, \quad 30, \quad 45, \quad 45, \quad 35.$$

   Find

   (a) the mean of the sample data;
   Solution: $\overline{x} = \frac{25+35+45+50+55+35+30+45+45+35}{10} = 40$.

   (b) the **sample** variance of the sample;
   Solution:
   $s^2 = \frac{(25-40)^2+(35-40)^2+(45-40)^2+(50-40)^2+(55-40)^2+(35-40)^2+(30-40)^2+(45-40)^2+(45-40)^2+(35-40)^2}{9} = 88.88\ldots$.

   (c) the **sample** standard deviation of the data.
   Solution: $s = \sqrt{s^2} = 9.428\ldots$.

4. A confectioner makes two raisin-nut mixtures. A Standard box contains 6oz of peanuts, 1oz of raisins and 4oz of cashews, and sells for \$4.25. A Deluxe box contains 12oz of peanuts, 3oz of raisins and 2oz of cashews, and sells for \$6.55. The confectioner currently has in stock 5400oz of peanuts, 1200oz of raisins, and 2400oz of cashews.

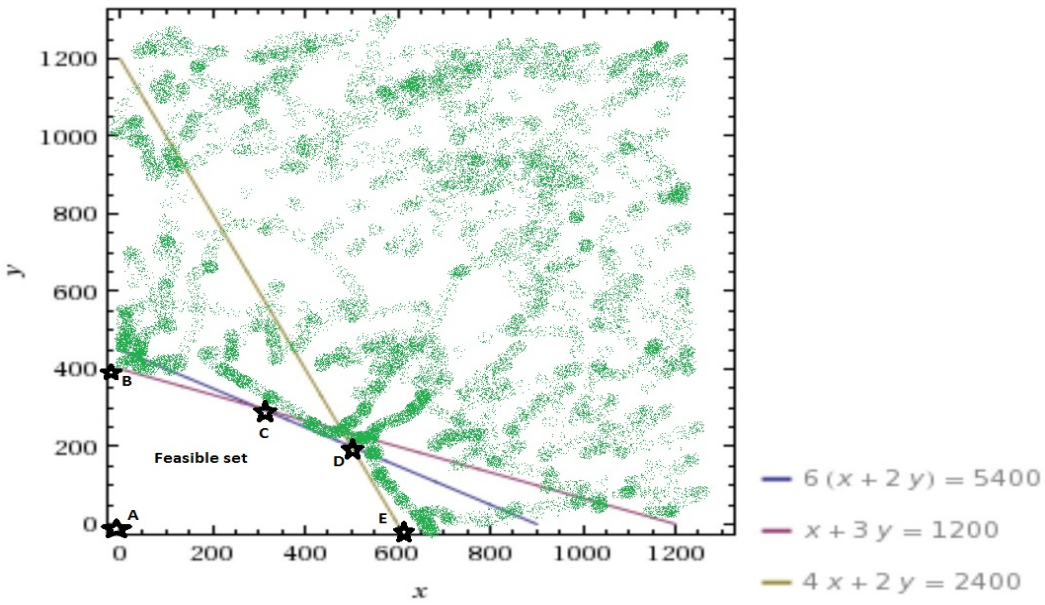
(a) Write down the linear programming problem that the confectioner has to solve, to decide how many Standard and how many Deluxe boxes to make, to maximize revenue (assuming he sells all boxes he makes).

Solution: Let S be number of Standard boxes made, and D the number of Deluxe. The LP is to maximize $4.25S + 6.55D$, subject to $6S + 12D \leq 5400$, $S + 3D \leq 1200$, $4S + 2D \leq 2400$ and $S, D \geq 0$.

(b) Show the feasible set on a graph.

Solution:

The feasible set is the 5-sided shape with corners A, B, C, D and E shown in the picture below (x-axis is S, y-axis is D):



(c) Find the corners of the feasible set.

Solution:

i. $A : (0, 0)$ [Objective value \$0]
ii. $B : (0, 400)$ [Objective value \$2,620]
iii. $C : (300, 300)$ [Objective value \$3,240]
iv. $D : (500, 200)$ [Objective value \$3,435]
v. $E : (600, 0)$ [Objective value \$2,550]

(d) Solve the linear programming problem.

Solution: From the calculations of the objective function at each corner of the feasible set, done in the last part, we see that the optimal solution is to make 500 boxes of Standard and 200 boxes of Deluxe (corner D).

(e) What is the confectioners optimal profit?

Solution: \$3,435.

(f) If the confectioner makes the mix of Standard and Deluxe boxes dictated by the solution to the linear programming problem, what inventory does he have left after he has made all the boxes?

Solution: At $S = 500$ and $D = 200$, he uses 5400oz of peanuts (so has none left), 1100oz of raisins (so has 100oz left) and 2400oz of cashews (so has none left). So: 100oz of raisins is all that is left.