

Math 10120 — Finite Mathematics

Some Counting Problems — with solutions

January 27, 2014

1. **The Hoosier Lottery:** When you buy a Powerball ticket, you select 5 white numbers from among the numbers 1 through 59, and one red number from among the numbers 1 through 35. How many different Powerball tickets can you buy? **Solution:** $C(59, 5) \times C(35, 1)$
2. **Poker hands:**
 - (a) A poker hand consists of a selection of 5 different cards from an ordinary deck of 52 cards. How many different poker hands are there? **Solution:** $C(52, 5)$
 - (b) An ordinary deck has 4 suits (hearts, clubs, spades, diamonds). In each suit there are 13 denominations (Ace, 2 through 10, Jack, Queen, King). How many poker hands involve cards from just a single suit? **Solution:** $4 \times C(13, 5)$ (choose suit, then five cards from that suit)
 - (c) Four-of-a-kind is a poker hand consisting of four cards of the same denomination, and one of another denomination (e.g., Ace of hearts, clubs, diamonds and spades, and five of hearts). How many four-of-a-kinds are there? **Solution:** 13×48 (choose denomination for the four, then one other card)
3. **Notre Dame Hockey:** Notre dame hockey has a 26-man roster, of which exactly three are goaltenders. Each game day, Coach Jeff Jackson has to choose
 - 21 players to suit up, exactly two of whom must be goaltenders
 - 6 of those players to start, exactly one of whom must be a goaltender
 - the order of pre-game announcement of the six starters, with the goaltender announced last.

How many options does he have in total? **Solution:** Lots of different symbolic answers, all leading to the same numerical value. For example, $C(3, 2) \times C(23, 19) \times C(2, 1) \times C(19, 5) \times 5 \times 4 \times 3 \times 2 \times 1 \times 1$

4. **Tossing a coin:**

- (a) I toss a coin 15 times, and record whether I got a head (H) or a tail (T) each time. How many different total outcomes could I record?

Solution: 2^{15}

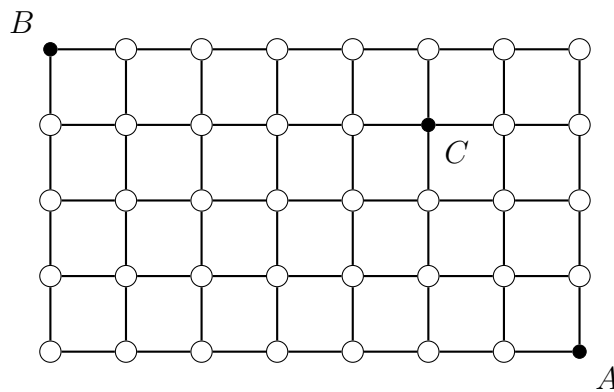
- (b) In how many of these outcomes do I record between 5 and 10 tails?

Solution: $C(15, 5) + C(15, 6) + C(15, 7) + C(15, 8) + C(15, 9) + C(15, 10)$

- (c) In how many of these outcomes do I record at least 2 tails? **Solution:**

Either $2^{15} - C(15, 0) - C(15, 1)$, or $C(15, 2) + C(15, 3) + C(15, 4) + C(15, 5) + C(15, 6) + C(15, 7) + C(15, 8) + C(15, 9) + C(15, 10) + C(15, 11) + C(15, 12)$

5. **Traveling through Manhattan:**



- (a) How many ways to walk from A (first and first) to B (fifth and eighth), if you only walk west and north (so always walk 11 blocks, with each block taking you closer to your final destination)? **Solution:** Either $C(11, 4)$ or $C(11, 7)$ (these numbers are equal)

- (b) How many ways to walk from A to B , if you have to pass through C on the way (and again you only walk west and north)? **Solution:** Lots of different symbolic answers, all leading to the same numerical value. For example, $C(5, 2) \times C(6, 5)$

6. **Picking a delegation:** A small company has three offices. The South Bend office has 12 employees, the Elkhart office has 18 and the Mishawaka office has 16 employees.

- (a) In how many ways can the company choose 4 people to go to a conference? **Solution:** $C(46, 4)$
- (b) How about if all the delegates to the conference must be from the same office? **Solution:** $C(12, 4) + C(18, 4) + C(16, 4)$
- (c) What about if all three offices must have at least one person going to the conference? **Solution:** $C(12, 2)C(18, 1)C(16, 1) + C(12, 1)C(18, 2)C(16, 1) + C(12, 1)C(18, 1)C(16, 2)$