Name: _____

MATH 10120 Finite Mathematics
Final Exam
May 7, 2014

Instructor: *Galvin/Diaz*

- Be sure that you have all 16 pages of the exam.
- The exam lasts for 2 hrs.
- There are 30 multiple choice questions, each worth 5 points.
- You may use a calculator.
- The Honor Code is in effect for this exam.
- There is a table of areas under the standard normal curve at the end of the exam.

May the force be with you!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	26.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	27.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	28.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)	29.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)	30.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)						
16.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1. (5 pts.) Let A and B be sets such that $n(B) = 12$, $n(A \cap B) = 8$ and $n(A \cup B) = 21$. Find $n(A)$.

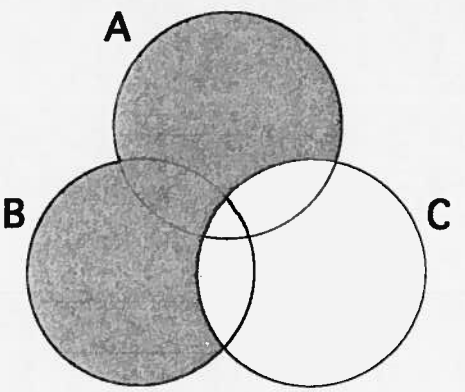
(a) 13 (b) 17 (c) 41 (d) 1 (e) 9

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$21 = n(A) + 12 - 8$

$21 - 4 = n(A) = 17$

2. (5 pts.) Which of the following corresponds to the area shaded in gray in the following Venn diagram? (Remember that the notation $(\ )'$ refers to the complement of the set $(\ )$).



Region is outside C but inside $A \cup B$

$(A \cup B) \cap C'$

(a) $(A \cup B)' \cap C$ (b) $(A \cap B) \cap C$ (c) $(A \cap B)' \cup C$

(d) $(A \cup B) \cap C'$ (e) $(A \cap B) \cap C'$

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3. (5 pts.) Let $A = \{a, b, c, d, e, f, g, h, i\}$. Not including the empty set and A itself, how many subsets does A have?

(a) 524 (b) 484 (c) 7 (d) 254 ~~(e)~~ 510

A has 9 elements
it has 2^9 subsets including A and $\emptyset$
Therefore it has $2^9 - 2 = 510$ such subsets

4. (5 pts.) Let M be the set of Notre Dame students enrolled in a math class, let B be the set of Notre Dame students enrolled in a biology class, and let E be the set of Notre Dame students enrolled in an engineering class. Let the universal set, U, be the set of all Notre Dame students. In words, what is $(M \cup B)' \cap E$?

(a) Notre Dame students **not** enrolled in engineering **or** math **or** biology

(b) Notre Dame students enrolled in engineering and **either** math **or** biology but **not** both

(c) Notre Dame students enrolled in engineering **and** math **and** biology

~~(d)~~ Notre Dame students enrolled in engineering but **not** math and **not** biology.

(e) Notre Dame students enrolled in engineering

$M \cup B$ = all students enrolled in Math or Biology or both
$(M \cup B)'$ = all students not enrolled (in Math or Biology or both)
= all " enrolled in neither math nor biology
= " " not enrolled in Math and not enrolled in Biology
$(M \cup B)' \cap E$ = all students enrolled in Engineering but not in math and not in biology

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5. (5 pts.) This exam contains 30 multiple choice problems, each of which has five choices. In how many ways can the exam be answered?

(a) 30^5 (b) 5^{30} (c) 30×5 (d) $P(30,5)$ (e) $C(30,5)$

By mult. principle

step 1: pick ans to Q 1 (5 ways)
" 2: " " " Q2 (5 ways)

⋮ ! ⋮

step 30: pick ans. to Q30 (5 ways)

Total = 5^{30} ways.

6. (5 pts.) Three couples go to a movie theatre. They sit in consecutive seats such that each couple is seating together, that is, each person is seating next to his/her partner. If there are 6 seats available, in how many ways can they sit?

(a) 24 (b) 48 (c) 720 (d) 8 (e) 1

Couple 1 Couple 2 Couple 3.

Mult. Principle

Step 1: Pick couple 1 (3ways)
" 2 arrang them (2ways)
Step 3 Pick couple 2 (2ways)
Step 4 arrange them (2ways)
step 5 pick couple 3 (1way)
step 6 arrang them (2ways)

Total = $3!(2^3) = 48$ ways

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7. (5 pts.) The 10 members of the "The Shirt" committee have to choose one president and two vice presidents (these have to be three different people). In how many ways can they choose these officers? [Note: here is no distinction between the two vice presidents; there isn't a "first" VP and a "second" VP]

(a) 36 (b) 360 (c) 45 (d) 120 (e) 720

Mult. Principle

Step 1: choose president (10 ways)

Step 2; choose 2 V.P.'s ($C(9,2)$ ways)

$= 10 \cdot \frac{9 \cdot 8}{2 \cdot 1} = 10 \cdot 36 = 360$ ways

8. (5 pts.) A university wants to assign a three digit number to each classroom of a new building. They can use the digits $\{1, 2, 3, 4, 5\}$ but they **cannot** use any digit more than once. How many classroom numbers can they assign if the numbers have to be **less than** 250?

(a) 10 (b) 24 (c) 60 (d) 12 (e) 21

classroom numbers

= # cl.#'s between 100 and 199 + # cl.#'s between 200 and 249

(1st digit = 1) (1st Digit = 2)

$\Rightarrow$ 1 _ _ (4 choices, 3 choices)

2 _ _ (3 choices, 3 choices)

$= 4 \cdot 3 + 3 \cdot 3 = 12 + 9 = 21$

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9. (5 pts.) The University wants to select 4 students for a feedback survey. They want all four students from either Prof. Galvin's section or Prof. Diaz's section. Prof. Galvin has 40 students in his class and Prof. Diaz has 30 students in his class. How many selections are possible?

(a) 2,851,080 (b) 118,795 (c) 2,504,542,950

(d) 22,005,480 (e) 916,895

samples = # samples from Galvin's section
+ # " " Diaz's section

$= C(40,4) + C(30,4)$

$= 118,795$

10. (5 pts.) The sample space of an experiment is $\{1,2,3,4,5\}$. The probability that the outcome is an even number is 0.4, the probability that the outcome is 1 is 0.25 and the probability that the outcome is 5 is 0.15. What is the probability that the outcome is 3?

(a) 0.2 (b) 0.8 (c) 0.4

(d) 0.6 (e) Cannot be determined with the given information.

	Prob
1	$x_1 = .25$
2	x_2
3	x_3
4	x_4
5	$x_5 = .15$

$x_2 + x_4 = .4$

$P(3) = 1 - P(\{1,2,4,5\})$

$= 1 - [P(1) + P(\{2,4\}) + P(5)]$

$= 1 - [.25 + .4 + .15]$

$= 1 - .8 = .2$

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11. (5 pts.) A dice is rolled twice and the numbers on the uppermost faces are recorded. Consider the three events:

E: At least one number is odd
F: exactly one of the numbers is 2
G: Both numbers are even

Which of the following are a pair of mutually exclusive events?

(a) E and G' (b) E and F' (c) F and G (d) E and G (e) E and F

$(1,1) \in E \cap G'$ $(1,1) \in E \cap F'$ $(2,2) \in F \cap G$ $E \cap G = \emptyset$ $(1,2) \in E \cap F$

12. (5 pts.) A dice is rolled three times. What is the probability that either the first roll is a six, or both of the last two rolls are sixes?

(a) $\frac{41}{216}$ (b) $\frac{6}{216}$ (c) $\frac{36}{216}$ (d) $\frac{42}{216}$ (e) $\frac{1}{216}$

$P(1^{st} \text{ six} \cup \text{both of last 2 six}) = P(1^{st} \text{ six}) + P(2^{nd} \text{ and } 3^{rd} \text{ are } 6) - P(\text{all 3 are } 6)$

$= \frac{1}{6} + \frac{1}{6 \cdot 6} - \frac{1}{6^3}$

$= \frac{6^2 + 6 - 1}{6^3} = \frac{41}{216}$

Could also use a Tree diagram

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13. (5 pts.) Three people are shooting at a target. The probabilities that they hit the target are 0.5, 0.6, and 0.8, respectively. Find the probability that they all miss the target.

~~(a)~~ 4% (b) 1% (c) 96% (d) 76% (e) 24%

Assuming that the events that they each hit the target ARE independent

$P(\text{all 3 miss}) = P(1^{st}\text{ Misses}) \cdot P(2^{nd}\text{ Misses}) P(3^{rd}\text{ Misses})$

$= (.5)(.4)(.2) = .04 = 4\%$

14. (5 pts.) A wallet contains seven \$1 bills, three \$5 bills, and five \$10 bills. A bill is selected at random from the wallet. Find the probability that the bill is a \$1 bill given that it is not a \$10 bill.

(a) $\frac{7}{15}$ (b) $\frac{4}{15}$ (c) $\frac{7}{12}$ ~~(d)~~ $\frac{7}{10}$ (e) $\frac{4}{5}$

$P(\$1 \mid \text{Not } \$10)$

$= \frac{P(\$1 \text{ and Not } \$10)}{P(\text{Not } \$10)} = \frac{P(\$1)}{P(\text{Not } \$10)} = \frac{7/15}{10/15}$

$= \frac{7}{10}$

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15. (5 pts.) Visiting the neonatal wing of the hospital, Dr. Jones noticed that a set of triplets had been born the night before. Asking about them, she was told that at least two of the babies were boys. Given this information, what is the probability that all three are boys? [Assume that within a set of triplets, each baby is equally likely to be a boy or a girl, independently of the sex of the others.]

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{3}{4}$

$$P(3B \mid \text{at least } 2B) = \frac{P(3B \cap (\text{at least } 2B))}{P(\text{at least } 2B)} = \frac{P(3B)}{P(\text{at least } 2B)}$$

$$= \frac{1/8}{P(\text{Ex } 2B) + P(3B)} = \frac{1/8}{3/8 + 1/8} = \boxed{\frac{1}{4}}$$

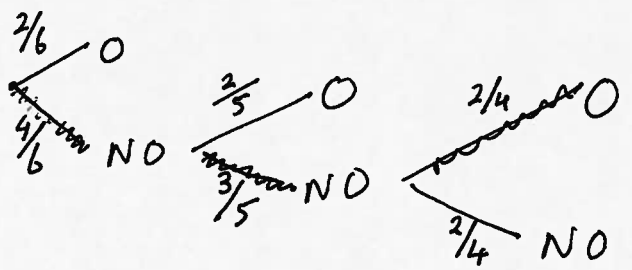
Boys is a binomial R.V. with $n = 3$, $\phi = 1/2$.

$$P(\text{Ex } 2B) = C(3,2)\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^1 = \frac{3}{8}$$

16. (5 pts.) I have 6 keys on my keyring. Two of them open my office door, the other four do not. I try to open my office door using randomly selected keys from my keyring, never trying the same key twice. What is the probability that I succeed in opening the door using the third key that I try?

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{8}$ (d) $\frac{1}{3}$ (e) $\frac{4}{27}$

Can use a Tree Diagram. O = key opens Door
NO = " Does not open Door



$$P(O \text{ on } 3^{rd} \text{ Try}) = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{1} = \frac{1}{5}$$

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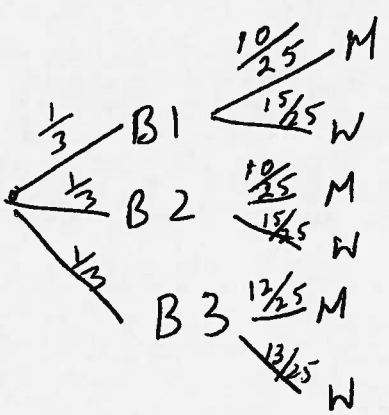
17. (5 pts.) Three buses, B1, B2 and B3, arrive at the South Bend Transpo Station at the same time. B1 arrives with 10 men and 15 women; B2 arrives with 10 men and 15 women; B3 arrives with 12 men and 13 women. A passenger is chosen at random. What is the probability that the passenger was in B3, given that he is a man?

Bayes Theorem!

(a) 0.48 (b) 0.16 (c) 0.426 (d) 0.375 (e) 0.315



$$P(B3|M) = \frac{P(B3 \cap M)}{P(M)}$$

$$= \frac{\frac{1}{3} \cdot \frac{12}{25}}{\frac{1}{3}\cdot\frac{10}{25} + \frac{1}{3}\cdot\frac{10}{25} + \frac{1}{3}\cdot\frac{12}{25}}$$

$$= \frac{\frac{12}{75}}{32/75} = \frac{12}{32} = .375$$

18. (5 pts.) The number of customers waiting in line at the express checkout of Martins Supermarket was counted at the beginning of each 3-min interval between 9a.m. and noon on Saturday. The data is as follows:

# customers	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	2	7	14	8	10	6	3	4	1

Total = 60

What is the relative frequency of the outcome "5 customers"?

(a) $\frac{5}{55}$ (b) $\frac{5}{60}$ (c) $\frac{8}{55}$ (d) $\frac{8}{60}$ (e) $\frac{2}{60}$

$$= \frac{\text{Freq}}{\text{Total \# observations}}$$

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19. (5 pts.) A game consists of rolling a die. If the number that shows up is even, the player wins, in dollars, the number shown. If the number that shows up is odd, the player loses, in dollars, the number shown. What is the expected value for the amount of money the player would win? [Note: A negative number means the player loses money.]

(a) \$ 0.5 (b) \$0 (c) −\$ 3.5 (d) −\$ 0.16 (e) \$ 3

Let X denote the player's earnings
The prob. dist. for X is
shown on the right.

X	P(X)	X P(X)
−1	1/6	−1/6
+2	1/6	2/6
−3	1/6	−3/6
+4	1/6	4/6
−5	1/6	−5/6
+6	1/6	+6/6
		3/6 = 1/2 = E(X).

20. (5 pts.) In a collection of 10 electronic components, three are defective. Two are selected at random and the number of defective components is noted. Let X be the number of defective components. Compute the probability distribution of X.

(a)

X	$P(X)$
0	3/50
1	21/50
2	21/50

(b)

X	$P(X)$
0	0
1	3/10
2	9/100

(c)

X	$P(X)$	
0	21/45	✓
1	21/45	✓
2	3/45	✓

(d)

X	$P(X)$	
0	21/45	✓
1	21/45	✓
2	6/45	✗

(e)

X	$P(X)$
0	21/50
1	21/50
2	8/50

10 components

7ND	3D

Sample size 2.

$P(X=0) = \frac{\#\text{ samples with 0D}}{\text{Total \# samples.}} = \frac{C(7,2)}{C(10,2)} = \frac{21}{45}$

$P(X=2) = \frac{\#\text{ samples with 2D}}{\text{Total \# samples}} = \frac{C(3,2)}{45} = \frac{3}{45}$

$P(X=1) = \frac{\#\text{ samples with 1D and 1ND}}{\text{Total \# samples}} = \frac{C(7,1)C(3,1)}{45} = \frac{21}{45}$

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21. (5 pts.) The table below gives partial information about the probability distribution of the random variable X, which has mean 2.

k	0	1	2	3	4
$P(X=k)$	.3	.1	.1	?	.2

What is the probability that X takes a value that is within one standard deviation of its mean?

(a) 1 (b) .2 (c) .5 (d) .1 (e) .4

First we find $P(x=3) = 1-(.3+.1+.1+.2) = 1-(.7) = .3$

Next we calculate S. Dev.

x	0	1	2	3	4
$P(x=x)$	.3	.1	.1	.3	.2
$x-\mu$	-2	-1	0	1	2
$(x-\mu)^2$	4	1	0	1	4
$(x-\mu)^2 P(x)$	1.2	.1	0	.3	.8

$\sigma^2 = 2.4$

$\sigma = \sqrt{2.4} \approx 1.55$

Want $P(2-1.55 \le X \le 2+1.55)$

$= P(.45 \le X \le 3.55)$

$= P(X \in \{1, 2, 3\})$

$= .1+.1+.3$

$= .5.$

22. (5 pts.) The most popular color for compact/sports cars is silver, with 20% of owners preferring that color. If six compact/sports car owners are selected at random, find the probability that **2 or more** of them prefer silver.

(a) $1-6(0.2)^1(0.8)^5$ (b) $(0.8)^6+6(0.2)^1(0.8)^5+15(0.2)^2(0.8)^4$

(c) $1-(0.8)^6-6(0.2)^1(0.8)^5$ (d) $15(0.8)^2$

(e) $15(0.8)^2(0.2)^4$

n = # selected = 6

p = prob. Like silver = .2.

X = # who Like silver in sample

was a binom. prob dist.

$P(X \ge 2) = 1-P(X<2)$

$= 1-[P(x=0)+P(x=1)]$

$= 1-[C(6,0)(.2)^0(.8)^6 + C(6,1)(.2)^1(.8)^5]$

$= 1-(.8)^6 - 6(.2)^1(.8)^5$

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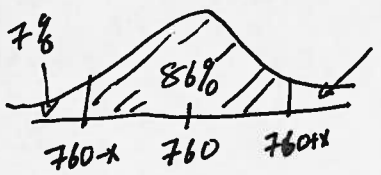
23. (5 pts.) Suppose that on a certain standardized test given nationally, the mean score is 760 with a standard deviation of 40. A statistician reports that 86% of all people taking the test received a score that was within x points of the mean. What is x (rounded to the nearest whole number)?

(a) 59 (b) 43 (c) 62 (d) 40 (e) 118

Lets assume that the test scores, X, are normally distributed with $\mu = 760$ and $\sigma = 40$.

We want a value x so that $P(760 - x \le X \le 760 + x) = .86$.



i.e. $P(X \le 760 + x) = .93$

using Z-scores we want x so that

$P\left(Z \le \frac{760 + x - 760}{40}\right) = .93$

or $P\left(Z \le \frac{x}{40}\right) = .93$.

Using the tables, we find $\frac{x}{40} \simeq 1.47$

And $x \simeq (1.47)40 \simeq 59$

24. (5 pts.) A dice is rolled 7 times. Let X be the number of sixes that come up. Estimate $P(2 \le X \le 4)$ using the **normal distribution**.

(a) 20% (b) 37% (c) 18% (d) 50% (e) 33%

X is a binomial R.V. with $n = 7$, $p = 1/6$.

Using the Normal approx to the binomial with mean $\mu = \frac{7}{6}$

and st. dev. $\sigma = \sqrt{npq} = \sqrt{\frac{35}{36}}$

We want to calculate $P(1.5 \le X \le 4.5)$

= (using calculator) normalcdf $\left(1.5, 4.5, \frac{7}{6}, \sqrt{\frac{35}{36}}\right)$

$= .367$

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25. (5 pts.) A student earns \$13 per hour working for the Computer Center's help line and \$9.50 per hour working at the Copy Center. Because of her course load, she limits her work to 25 hours per week. She wants to earn at least \$275 each week. Express this information as linear inequalities.

(a) $13x + 9.5y \leq 275$, $x + y \geq 25$

(b) $13x + 9.5y \leq 275$, $x + y \leq 25$, $x \geq 0, y \geq 0$

(c) $13x + 9.5y \geq 275$, $x + y \leq 25$

(d) $13x + 9.5y \geq 275$, $x + y \leq 25$, $x \geq 0, y \geq 0$

(e) $13x + 9.5y \geq 275$, $x + y \geq 25$, $x \geq 0, y \geq 0$

LET x = # hrs. spent working @ comp. con.

LET y = # " " " @ copy cen.

	x Comp.	y Copy.
Earn per hr.	13	9.5

WANT:

$13x + 9.5y \geq 275$

$x + y \leq 25$

MUST HAVE: $x \geq 0$, $y \geq 0$

26. (5 pts.) Find the minimum of the objective function $4x + 7y$ on the feasible set given below:

MIN. (If it exists) MUST occurr At A, B or C

A = y-int. of $y = 30 - 2x$

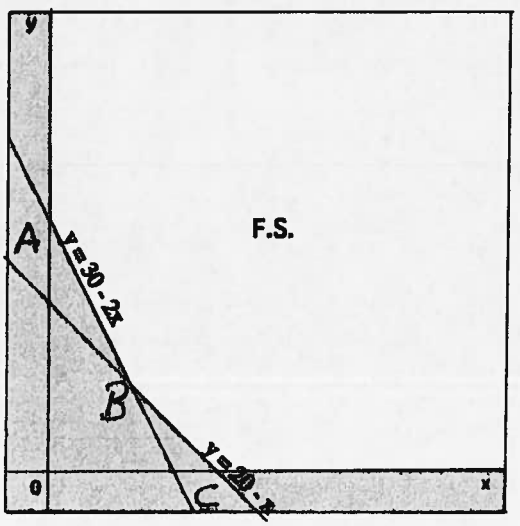
when $x = 0 \rightarrow y = 30$

A(0, 30)

C = x-int of $y = 20 - x$

$y = 0 \rightarrow x = 20$

C(20, 0)



B: $y = 30 - 2x$ meets $y = 20 - x$

$\rightarrow 30 - 2x = 20 - x$.

$\rightarrow 10 = x$

$\Rightarrow y = 20 - x = 10$

B(10, 10)

Vertex	$4x + 7y$.
(0, 30)	210
(20, 0)	80 ← MIN.
(10, 10)	40 + 70 = 110

(a) 70 (b) 250 (c) 80 (d) 110 (e) 210

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27. (5 pts.) Raphael (R) and Chris (C) play the rock-paper-scissors game. Nothing happens if they both show the same shape. If one player chooses rock and the other chooses scissors, then the player who chose rock wins \$5. If one player chooses scissors and the other chooses paper, then the player who chose scissors wins \$3. If one player chooses paper and the other chooses rock, then the player who chose paper wins \$1. Find the pay-off matrix for Raphael (R). [Note: as usual, the rows indicate R's choices and the columns indicate C's choices.]

(a) ✓

	Rock	*Paper*	*Scissors*
Rock	0	−1	5 ✓
Paper	1	0	−3 ✓
Scissors	−5	3	0

(b) ✗

	Rock	*Paper*	*Scissors*
Rock	0	−5	1 ✗
Paper	5	0	3
Scissors	−1	−3	0

(c) ✗

	Rock	*Paper*	*Scissors*
Rock	0	5	−1 ✗
Paper	−5	0	3
Scissors	1	−3	0

(d) ✗

	Rock	*Paper*	*Scissors*
Rock	0	−3	5 ✓
Paper	3	0	−1 ✗
Scissors	−5	1	0

(e) ✗

	Rock	*Paper*	*Scissors*
Rock	0	1	−5 ✗
Paper	−1	0	3
Scissors	5	−3	0

R(Rock) C(Sci) → R gets \$5

R(Paper) C(Sci) → R Loses \$3

This eliminates every answer but (a). You should check that the other entries are also correct.

28. (5 pts.) Rosita (R) and Carlos (C) play a zero-sum game with pay-off matrix for Rosita given by:

	C1	*C2*	Min
R1	1	3	1
R2	5	2	2
	5	3	
	May		

No S.P. ⇒ NOT ST. Determined. (d) is false.

Which of the following statements is **FALSE**?

(a) If both players play each of their options equally likely, on average Rosita wins 2.75 True.

(b) If Rosita is equally likely to play each of her options, then it is better for Carlos to play *C*2 than *C*1 ✓

(c) If Carlos plays *C*2, then it is better for Rosita to play *R*1 than *R*2 True. ✓ $(r_1\ r_2)\begin{pmatrix}1 & 3\\ 5 & 2\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = (r_1\ r_2)\begin{pmatrix}3\\ 2\end{pmatrix}$

(d) This is a strictly determined game ✗ (3 ↓ 2 ↓)

(e) The entry in the row 1, column 2 position of the payoff matrix is not a saddle point TRUE (NO S.P.'s).

Rosita plays both options with Prob. .5

Ex Pay-off. For R. $= (.5\ \ .5)\begin{pmatrix}1 & 3\\ 5 & 2\end{pmatrix}\begin{pmatrix}C_1\\ C_2\end{pmatrix} = (3\ \ 2.5)\begin{pmatrix}C_1\\ C_2\end{pmatrix}$

(a) True $(3\ \ 2.5)\begin{pmatrix}.5\\ .5\end{pmatrix} = 2.75$

(b) True. C → (0) R's Ex pay-off. = 2.5 ← better for C. C → (1) R's Ex pay-off = 3

Name: _____

Instructor: *Galvin/Diaz*

29. (5 pts.) Rob (R) and Chad (C) play a zero-sum, two person game. The payoff matrix for the game is:

$$\begin{bmatrix} 4 & -3 \\ -2 & 3 \end{bmatrix}$$

If Rob (playing rows) uses the strategy $\begin{bmatrix} 0.4 & 0.6 \end{bmatrix}$ and Chad uses the strategy $\begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$, what is the (expected) value of the game?

(a) 0.06 (b) 0.36 (c) 0.16 (d) 0 ~~(e)~~ 0.46

$$\begin{bmatrix} .4 & .6 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .4 & .6 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} = .46$$

30. (5 pts.) Rusty (R) and Crusty (C) play the following game: they both shout out a number, either 1 or 2. If they both shout out 1, Rusty wins 2 points. If they both shout out 2, Crusty wins 1 points. If Rusty says 1 and Crusty says 2, Rusty wins 1 point, and if Rusty says 2 and Crusty says 1, Rusty wins 5 points. If Crusty plays the strategy $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, which of the following is the best counter-strategy for Rusty?

~~(a)~~ [1 0] (b) [.2 .8]

(c) [0 1] (d) [.5 .5]

(e) Rusty has no good counter-strategy

Pay-off matrix $= \begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}$

Best counterstrategy is always a pure strategy [1 0] or [0 1]

Expected Pay-off for R with strategy $[r_1, r_2]$ is

$$\begin{bmatrix} r_1 & r_2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = r_1 - r_2.$$

Best c. strategy for R is [1, 0]