Finite Mathematics (Math 10120), Spring 2016

Quiz 4, Wednesday March 23

Solutions

1. (5 pts) I play a casino game in which I play \$2 to pay. I have a probability .1 of getting my \$2 back and also winning \$3; I have a probability .2 of getting my \$2 back and also winning \$1; I have a probability .3 of getting my \$2 back and winning nothing else; and I have a probability .4 of losing my \$2.

   Let Y be my net winnings in a play of this game. Calculate the expected value and the variance of Y.

   Solution: Here is the probability distribution table of Y:

y	$\mathbf{P}(Y = y)$
3	.1
1	.2
0	.3
-2	.4

   We can use the following table to calculate $E(Y)$ and $E(Y^2)$:

y	$\mathbf{P}(Y = y)$	$y\mathbf{P}(Y = y)$	y^2	$y^2\mathbf{P}(Y = y)$
3	.1	.3	9	.9
1	.2	.2	1	.2
0	.3	0	0	0
-2	.4	$-.8$	4	1.6
		$E(Y) = -.3$		$E(Y^2) = 2.7$

   So the expected value of Y is $-.3$ and the variance is $E(Y^2) - E(Y)^2 = 2.7 - (-.3)^2 = 2.61$.

2. (5 pts) I've just gotten off a flight from Chicago to South Bend. There are four bags in the hold, two of which are mine. The bags will come out on the luggage carousel in a random order. Let X be the number of bags that have come out the moment the second of my bags comes out. For example, if the first two bags out are mine then $X = 2$; if mine are the last two to come out then $X = 4$. The other possible value for X is 3.

   Find the probability distribution of X. That is, find $\mathbf{P}(X = 2)$, $\mathbf{P}(X = 3)$ and $\mathbf{P}(X = 4)$. **Hint**: draw a tree diagram, branching on whether the first bag, second bag, third bag etc. is one of mine or not.

x	$\mathbf{P}(X = x)$
2	$1/6 \approx .167$
3	$2/6 = 1/3 \approx .333$
4	$3/6 = 1/2 = .5$

   Solution: Here are all the possibilities for this experiment:

   - One of my bags first, one of my bags second, $X = 2$, probability $(2/4)(1/3) = 1/6$.
   - One of my bags first, one of the other bags second, one of my bags third, $X = 3$, probability $(2/4)(2/3)(1/2) = 1/6$
   - One of my bags first, one of the other bags second, one of the other bags third, one of my bags fourth, $X = 4$, probability $(2/4)(2/3)(1/2)(1/1) = 1/6$

- One of the other bags first, one of my bags second, one of my bags third, $X = 3$, probability $(2/4)(2/3)(1/2) = 1/6$
- One of the other bags first, one of my bags second, one of the other bags third, one of my bags fourth, $X = 4$, probability $(2/4)(2/3)(1/2)(1/1) = 1/6$
- One of the other bags first, one of the other bags second, one of my bags third, one of my bags fourth, $X = 4$, probability $(2/4)(1/3)(2/2)(1/1) = 1/6$

In one of the six equally-likely scenarios $X = 2$; in two of them $X = 3$ and in three of them $X = 6$. That leads to the probability distribution shown above. Notice that the sum of the three assigned probabilities is 1!