A puzzle about Roulette

Roulette seems like a fool's game. But here's a possible strategy for playing it:

1. Begin by betting a dollar on red.
2. If you win, take your winnings and go home.
3. If you lose, place two one-dollar bets in a row on red.
4. Whatever happens on those two rolls, go home (either with your winnings to date, or cutting your losses)

Question: Is this a winning strategy? Specifically, what is the probability that you will leave the Roulette wheel with more money than you began with, and is this probability more or less than 1/2? (Recall that in Roulette, the probability of winning on a single roll by betting on red is 18/38)

Solution

Let X be net winnings from this strategy. Possible outcomes/values for X:

- Win on first roll, probability $18/38 \approx .474$, $X = +1$
- Lose on first, win on next two, probability $(20/38)(18/38)^2 \approx .118$, $X = +1$
- Lose on first, win exactly one of next two, probability $(20/38)2(18/38)(20/38) \approx .262$, $X = -1$
- Lose all three, probability $(20/38)^3 \approx .146$, $X = -3$.

So X takes value $+1$ with probability $\approx .592$, value -1 with probability $\approx .262$, and value -3 with probability $\approx .146$. So the strategy *is* winning — you have a get gain more often than a net loss!