A strategy puzzler

I have five envelopes, exactly one of which contains a prize. Five people, one after another, come to me and take one of the envelopes (their choice, but without looking inside first!). They open it, and if it is empty they discard it (it can never be selected again). The game ends when someone selects the envelope with the prize.

If you are one of the five people playing this game, would you rather be early in the line of five players (so you have a high chance of getting to play, but a low chance of winning), late in the line (so there's a chance that the game will be over before it gets to your turn, but if it gets to your turn, you have a high chance of winning), or somewhere in the middle?

Solution

Let $A_1 =$ be the event that player 1 wins, with A_2, A_3, A_4 and A_5 defined similarly. We easily have $\mathbf{P}(A_1) = 1/5$.

For player 2 to win, it must be that first player 1 draws and looses and then player 2 draws and wins, so

$$\mathbf{P}(A_2) = \mathbf{P}(A_1^c \cap A_2) = \mathbf{P}(A_1^c)\mathbf{P}(A_2|A_1^c) = \frac{4}{5} \cdot \frac{1}{4} = 1/5.$$

Similarly,

$$\mathbf{P}(A_3) = \mathbf{P}(A_1^c)\mathbf{P}(A_2^c|A_1^c)\mathbf{P}(A_3|A_2^c \cap A_1^c) = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = 1/5,$$

$$\mathbf{P}(A_4) = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = 1/5,$$

and

$$\mathbf{P}(A_5) = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = 1/5.$$

So it doesn't matter when you draw — you always have the same probability $(1/5)$ of winning.