Expected Value and Variance

Have you ever wondered whether it would be "worth it" to buy a lottery ticket every week, or pondered questions such as "If I were offered a choice between a million dollars, or a 1 in 100 chance of a billion dollars, which would I choose?"

One method of deciding on the answers to these questions is to calculate the **expected** earnings of the enterprise, and aim for the option with the higher expected value.

This is a useful decision making tool for problems that involve repeating many trials of an experiment — such as investing in stocks, choosing where to locate a business, or where to fish.

(For once-off decisions with high stakes, such as the choice between a sure 1 million dollars or a 1 in 100 chance of a billion dollars, it is unclear whether this is a useful tool.)

Example

John works as a tour guide in Dublin. If he has 200 people or more take his tours on a given week, he earns €1,000. If the number of tourists who take his tours is between 100 and 199, he earns €700. If the number is less than 100, he earns €500. Thus John has a **variable** weekly income.

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From experience, John knows that he earns €1,000 fifty percent of the time, €700 thirty percent of the time and €500 twenty percent of the time. John's weekly income is a **random variable** with a probability distribution

Income	**Probability**
€1,000	0.5
€700	0.3
€500	0.2

Example

What is John's average income, over the course of a 50-week year?

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Over 50 weeks, we expect that John will earn

- €1000 on about 25 of the weeks (50%);
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$$\frac{25(€1000) + 15(€700) + 10(€500)}{50} = €810.$$

Dividing through by 50, this calculation changes to

$$.5(€1000) + .3(€700) + .2(€500) = €810.$$

Expected Value of a Random Variable

The answer in the last example stays the same no matter how many weeks we average over. This suggests the following: If X is a random variable with possible values $x_1, x_2, \ldots, x_n$ and corresponding probabilities $p_1, p_2, \ldots, p_n$, the **expected value** of X, denoted by $\mathbf{E}(X)$, is

$$\boxed{\mathbf{E}(X) = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n.}$$

Outcomes **X**	**Probability** $\mathbf{P}(X)$	**Out.** $\times$ **Prob.** $X\mathbf{P}(X)$
x_1	p_1	$x_1 p_1$
x_2	p_2	$x_2 p_2$
$\vdots$	$\vdots$	$\vdots$
x_n	p_n	$x_n p_n$
		Sum $= \mathbf{E}(X)$

Expected Value of a Random Variable

We can interpret the expected value as the long term average of the outcomes of the experiment over a large number of trials. From the table, we see that the calculation of the expected value is the same as that for the average of a set of data, with relative frequencies replaced by probabilities.

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Warning: The expected value really ought to be called the expected mean. It is NOT the **value** you most expect to see but rather the average (or mean) of the values you see over the course of many trials.

Coin tossing example

Flip a coin 4 times and observe the sequence of heads and tails. Let X be the number of heads in the observed sequence. Last time we found the following probability distribution for X:

X	$\mathbf{P}$(X)
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

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Find the expected number of heads for a trial of this experiment, that is find $\mathbf{E}(X)$.

$$\mathbf{E}(X) = \frac{1}{16} \cdot 0 + \frac{4}{16} \cdot 1 + \frac{6}{16} \cdot 2 + \frac{4}{16} \cdot 3 + \frac{1}{16} \cdot 4 = \frac{0+4+12+12+4}{16} = \frac{32}{16} = 2.$$

NFL example

The following probability distribution from "American Football" *Statistics in Sports, 1998*, by Hal Stern, has an approximation of the probabilities for yards gained on a running play in the NFL. Actual play by play data was used to estimate the probabilities. (-4 represents 4 yards lost on a running play).

x, yards	prob	x, yards	prob
-4	.020	6	.090
-2	.060	8	.060
-1	.070	10	.050
0	.150	15	.085
1	.130	30	.010
2	.110	50	.004
3	.090	99	.001
4	.070		

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$\mathbf{E}(X) = (-4)\cdot .020 + (-2)\cdot 0.060 + (-1)\cdot 0.070 + 0\cdot 0.150 + 1\cdot 0.130 + 2\cdot 0.110 + 3\cdot 0.090 + 4\cdot 0.070 + 6\cdot 0.090 + 8\cdot 0.060 + 10\cdot 0.050 + 15\cdot 0.085 + 30\cdot 0.010 + 50\cdot 0.004 + 99\cdot 0.001 = 4.024$

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(b) How much would you expect to win/lose if you bet \$1 on red 100 times? What would the casino expect to earn if you bet \$1 on red 100 times?

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You would expect to win $100 \cdot \mathbf{E}(X) = -200/38 \approx -\$5.26.$

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You would expect to win $100 \cdot \mathbf{E}(X) = -200/38 \approx -\$5.26.$

Your loss is the casino's gain so the casino's earnings are the negative of your loss: \$5.26.

A winning(?) strategy for Roulette?

Roulette seems like a fool's game. But here's a possible strategy for playing it:

1. Begin by betting a dollar on red.
2. If you win, take your winnings and go home.
3. If you lose, place two one-dollar bets in a row on red.
4. Whatever happens on those two rolls, go home (either with your winnings to date, or cutting your losses)

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Question: Is this a winning strategy? Specifically, what is the probability that you will leave the Roulette wheel with more money than you began with, and is this probability more or less than 1/2?

A winning(?) strategy for Roulette?

Let X be net winnings from this strategy. Possible outcomes/values for X:

- Win on first roll, probability $18/38 \approx .474$, $X = +1$
- Lose on first, win on next two, probability $(20/38)(18/38)^2 \approx .118$, $X = +1$
- Lose on first, win exactly one of next two, probability $(20/38)2(18/38)(20/38) \approx .262$, $X = -1$
- Lose all three, probability $(20/38)^3 \approx .146$, $X = -3$.

So X takes value $+1$ with probability $\approx .592$, value -1 with probability $\approx .262$, and value -3 with probability $\approx .146$. The strategy *is* winning — you have a net gain more often than a net loss!

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So X takes value $+1$ with probability $\approx .592$, value -1 with probability $\approx .262$, and value -3 with probability $\approx .146$. The strategy *is* winning — you have a net gain more often than a net loss!

On the other hand,
$\mathbf{E}(X) = 1(.592) - 1(.262) - 3(.146) = -.108$. So on *average*, playing this strategy long-term, you will lose money :(

Gambling example

The rules of a carnival game are as follows:

1. The player pays \$1 to play the game.
2. The player then flips a fair coin, if the player gets a head the game attendant gives the player \$2 and the player stops playing.
3. If the player gets a tail on the coin, the player rolls a fair six-sided die. If the player gets a six, the game attendant gives the player \$1 and the game is over.
4. If the player does not get a six on the die, the game is over and the game attendant gives nothing to the player.

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Let X denote the player's (net) earnings for this game.
Last time, we saw that X has probability distribution

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(a) What are the expected earnings for the player for each play of this game?

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(b) What are the expected earnings for the game host for each play of this game?

Host's earnings are negative of your earnings:
$-1/12 \approx -\$0.08$.

Variance and standard deviation

Let us return to the initial example of John's weekly income which was a random variable with probability distribution

Income	**Probability**
€1,000	0.5
€700	0.3
€500	0.2

with mean €810. Over 50 weeks, we might expect the **variance** of John's weekly earnings to be roughly

$$\frac{25(\text{€}1000\text{-}\text{€}810)^2 + 15(\text{€}700\text{-}\text{€}810)^2 + 10(\text{€}500\text{-}\text{€}810)^2}{50} = 49,900$$

or

$$.5(\text{€}1000\text{-}\text{€}810)^2 + .3(\text{€}700\text{-}\text{€}810)^2 + .2(\text{€}500\text{-}\text{€}810)^2 = 49,900$$

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$$\boxed{\textbf{Variance} = \boldsymbol{\sigma}^2(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \cdots + p_n(x_n-\mu)^2}$$

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x_i	p_i	$x_i p_i$	$(x_i-\mu)$	$(x_i-\mu)^2$	$p_i(x_i-\mu)^2$
x_1	p_1	$x_1 p_1$	$(x_1-\mu)$	$(x_1-\mu)^2$	$p_1(x_1-\mu)^2$
x_2	p_2	$x_2 p_2$	$(x_2-\mu)$	$(x_2-\mu)^2$	$p_2(x_2-\mu)^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	p_n	$x_n p_n$	$(x_n-\mu)$	$(x_n-\mu)^2$	$p_n(x_n-\mu)^2$
		Sum $=\mu$			**Sum** $=\boldsymbol{\sigma}^2(X)$

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Use the value for $\mu = \mathbf{E}(X)$ found above to find the variance and standard deviation of X, that is find $\boldsymbol{\sigma}^2(X)$ and $\boldsymbol{\sigma}(X)$.

Gambling example

$\mathbf{x}_i$	$\mathbf{p}_i$	$\mathbf{x}_i \cdot \mathbf{p}_i$	$(\mathbf{x}_i - \mu)$	$(\mathbf{x}_i - \mu)^2$	$\mathbf{p}_i \cdot (\mathbf{x}_i - \mu)^2$
-1	5/12	$\frac{-5}{12}$	$\frac{-13}{12}$	$\frac{169}{144}$	$\frac{845}{1728}$
0	1/12	$\frac{0}{12}$	$\frac{-1}{12}$	$\frac{1}{144}$	$\frac{1}{1728}$
1	6/12	$\frac{6}{12}$	$\frac{11}{12}$	$\frac{121}{144}$	$\frac{726}{1728}$
		$\textbf{Sum} = \mu = \frac{1}{12}$			$\textbf{Sum} = \boldsymbol{\sigma}^2(X) = \frac{1572}{1728} \approx 0.909$

$\boldsymbol{\sigma} \approx 0.953.$

Coin tossing example

An experiment consists of flipping a coin 4 times and observing the sequence of heads and tails. The random variable X is the number of heads in the observed sequence. Last time we found the following probability distribution for X:

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4	1/16

We saw above that the expected value for this random variable is $\mathbf{E}(X) = 2$. Find $\boldsymbol{\sigma}^2(X)$ and $\boldsymbol{\sigma}(X)$.

Coin tossing example

x_i	p_i	$x_i \cdot p_i$	$(x_i - \mu)$	$(x_i - \mu)^2$	$p_i \cdot (x_i - \mu)^2$
0	$\frac{1}{16}$	$\frac{0}{16}$	-2	4	$\frac{4}{16}$
1	$\frac{4}{16}$	$\frac{4}{16}$	-1	1	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	0	0	$\frac{0}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	1	1	$\frac{4}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	2	4	$\frac{4}{16}$
		Sum $= \mu = 2$			**Sum** $= \boldsymbol{\sigma}^2(X) = 1$

$\boldsymbol{\sigma} = 1.$

Another formula for variance

Using $(x-\mu)^2 = x^2 - 2\mu x + \mu^2$, we get another formula for variance:

$$
\begin{aligned}
\boldsymbol{\sigma}^2(X) &= p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \cdots + p_n(x_n-\mu)^2 \\
&= p_1(x_1^2 - 2\mu x_1 + \mu^2)^2 + \cdots + p_n(x_n^2 - 2\mu x_n + \mu^2)^2 \\
&= [p_1 x_1^2 + \cdots + p_n x_n^2] + \\
&\quad -2\mu[p_1 x_1 + \cdots p_n x_n] + \\
&\quad \mu^2[p_1 + \cdots p_n] \\
&= \mathbf{E}(X^2) - 2\mu\mathbf{E}(X) + \mu^2 \\
&= \mathbf{E}(X^2) - 2\mathbf{E}(X)\mathbf{E}(X) + \mathbf{E}(X)^2 \\
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&= \mathbf{E}(X^2) - 2\mu\mathbf{E}(X) + \mu^2 \\
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$$\boxed{\boldsymbol{\sigma}^2(X) = \mathbf{E}(X^2) - \mathbf{E}(X)^2}.$$

Redoing the coin example

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0	$\frac{1}{16}$	$\frac{0}{16}$	-2	4	$\frac{4}{16}$
1	$\frac{4}{16}$	$\frac{4}{16}$	-1	1	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	0	0	$\frac{0}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	1	1	$\frac{4}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	2	4	$\frac{4}{16}$
		Sum $= \mu = 2$			**Sum** $= \boldsymbol{\sigma}^2(X) = 1$

$\boldsymbol{\sigma} = 1.$

Redoing the coin example

Using the new formula:

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
0	$\frac{1}{16}$	$\frac{0}{16}$	0	$\frac{0}{16} = 0$
1	$\frac{4}{16}$	$\frac{4}{16}$	1	$\frac{4 \cdot 1}{16} = \frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	4	$\frac{6 \cdot 4}{16} = \frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	9	$\frac{4 \cdot 9}{16} = \frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	16	$\frac{1 \cdot 16}{16} = \frac{16}{16}$
		$\mathbf{Sum} = \mathbf{E}(X) = \mu = 2$		$\mathbf{Sum} = \mathbf{E}(X^2) = \frac{80}{16} = 5$

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1	$\frac{4}{16}$	$\frac{4}{16}$	1	$\frac{4 \cdot 1}{16} = \frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	4	$\frac{6 \cdot 4}{16} = \frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	9	$\frac{4 \cdot 9}{16} = \frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	16	$\frac{1 \cdot 16}{16} = \frac{16}{16}$
		$\mathbf{Sum} = \mathbf{E}(X) = \mu = 2$		$\mathbf{Sum} = \mathbf{E}(X^2) = \frac{80}{16} = 5$

Hence

$$\boldsymbol{\sigma}^2(X) = \mathbf{E}(X^2) - \mathbf{E}(X)^2 = 5 - 2^2 = 1$$

Back to the Roulette strategy

Recall that we had a Roulette strategy where your winnings X had probability distribution

$$P(X = 1) = .592, \quad P(X = -1) = .262, \quad P(X = -3) = .146.$$

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The strategy "bet once on Red" has expected winnings $-.053$, with standard deviation .998 — our complicated strategy is a little bit worse on average, and much more unstable