Feasible Sets

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The graph of the feasible set for a system of inequalities is the set of all points in the intersection of the graphs of the individual inequalities.

Constraints

Terminology: A linear inequality of any of the forms

$$a_0x + a_1y \leqslant b, \qquad a_0x + a_1y < b,$$

$$a_0x + a_1y \geqslant b, \qquad a_0x + a_1y > b,$$

where a_0, a_1 and b are constants, is called a **constraint** in an optimization. The corresponding **constraint line** is $a_0x + a_1y = b$. The restrictions $x \geqslant 0$, $y \geqslant 0$ are called **non-negativity conditions**.

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A system of constraint lines divides the plane into a bunch of regions. The feasible set will be one of these regions.

Determining the feasible set

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All methods start by drawing the constraint lines.

1. For each constraint inequality, decide which side of the constraint line satisfies the inequality. Take the intersection of each of the sets.
2. Pick a point in a region and see if it satisfies the inequality. If it does, the region containing this point is the feasible set. If not, pick a point in a different region. Continue until you find the feasible set. If you check all the regions and none work then the feasible set is empty.

Determining the feasible set

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Step 3. Pick another constraint line and divide $\mathbf{P}_1$ into those regions which satisfies the $>$ inequality and those which satisfies the $<$ inequality. The new "possible set" $\mathbf{P}_2$ is the subset of the previous "possible set" which satisfy the correct second inequality.

Determining the feasible set

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Step 4 Repeat step 3 until you have used all the constraint lines, getting $\mathbf{P}_3, \ldots, \mathbf{P}_n$. If at any time $\mathbf{P}_r$ is empty you are done and the feasible set is empty.

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After you have used all the constraint lines, the $\mathbf{P}_n$ will have one region left in it and this region is the feasible set.

Determining the feasible set

Example Determine if $(x, y) = (1, 2)$ is in the feasible set for the system of inequalities shown below:

$$2x + 3y \geqslant 6$$
$$2x - 3y \geqslant 15$$

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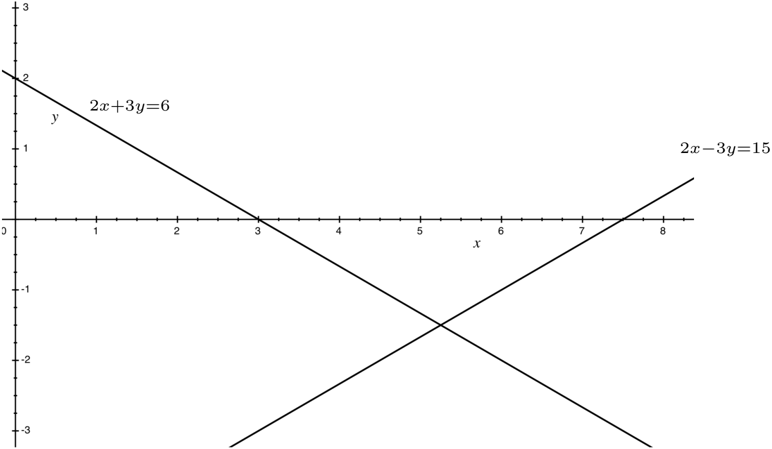
On the next few slides, the three methods for graphing the feasible set for a system of inequalities are illustrated, using this example.

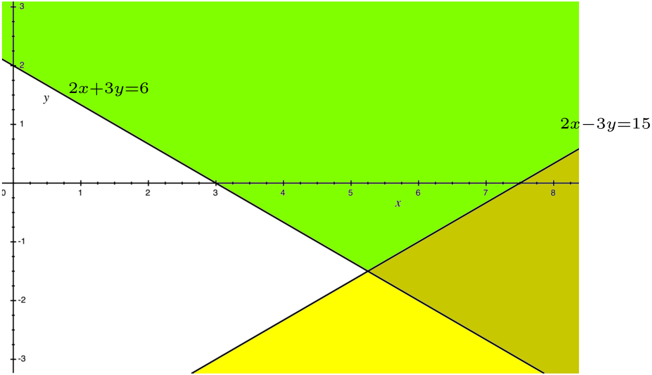
Determining the feasible set

The two lines divide the plane into four regions:



Determining the feasible set

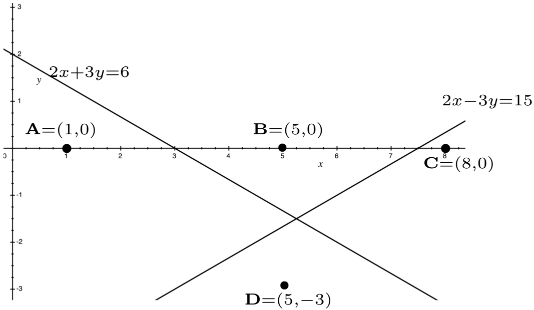
First method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



The green region is the half-plane satisfying the first inequality. The yellow region is the half-plane satisfying the second. The brown region, which is the intersection of the green and yellow regions, is the feasible set.

Determining the feasible set

Second method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



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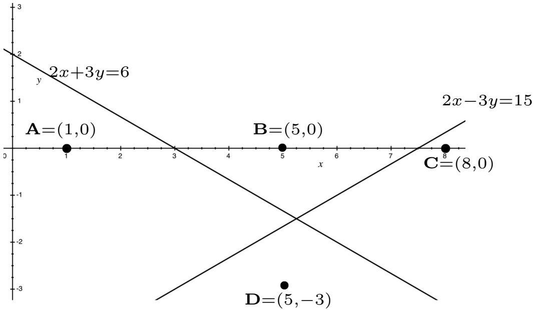

Region **A** includes $(1, 0)$, which satisfies neither inequality

Determining the feasible set

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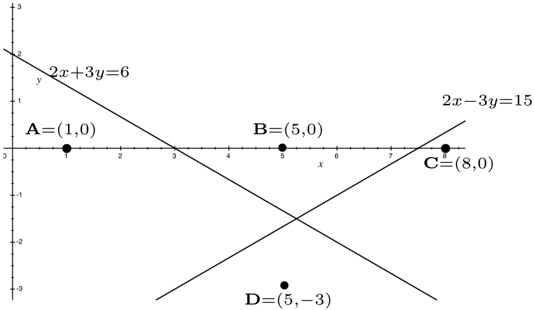
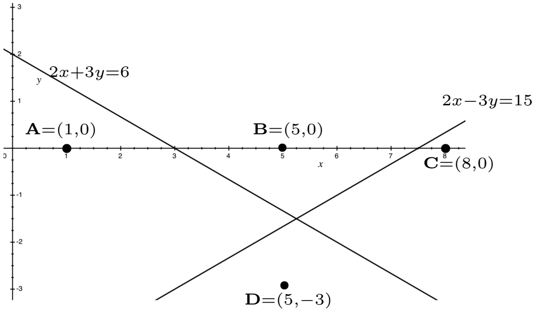


Region **A** includes $(1, 0)$, which satisfies neither inequality
Region **B** includes $(5, 0)$, which satisfies the first but not the second inequality

Determining the feasible set

Second method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



Region **A** includes $(1, 0)$, which satisfies neither inequality
Region **B** includes $(5, 0)$, which satisfies the first but not the second inequality
Region **C** includes $(7, 0)$, which satisfies both inequalities

Determining the feasible set

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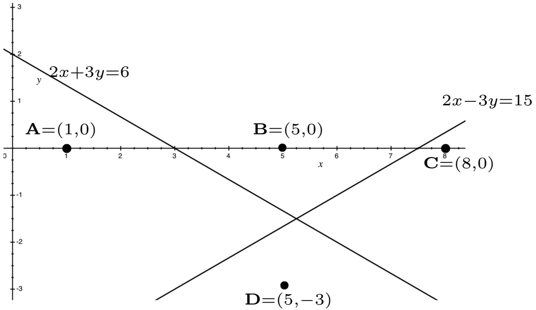
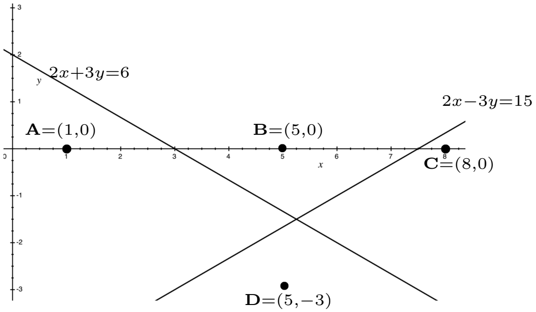


Region **A** includes $(1, 0)$, which satisfies neither inequality
Region **B** includes $(5, 0)$, which satisfies the first but not the second inequality
Region **C** includes $(7, 0)$, which satisfies both inequalities
Region **C** is the feasible set (no need to check **D**)

Determining the feasible set

Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



Determining the feasible set

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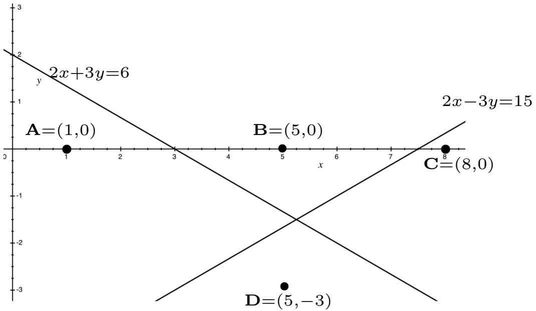
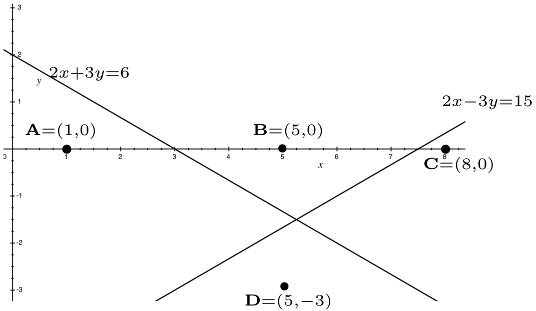


A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other.

Determining the feasible set

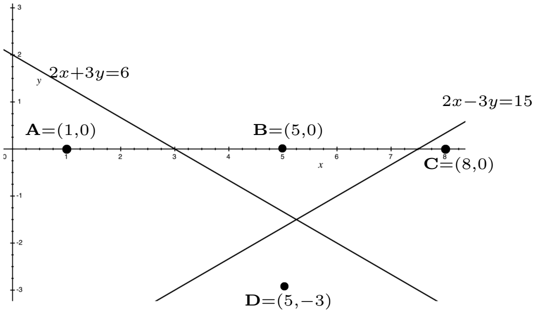
Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_1 = \{\mathbf{B}, \mathbf{C}\}$.

Determining the feasible set

Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_1 = \{\mathbf{B}, \mathbf{C}\}$.
B lies on one side of $2x - 3y = 15$ and **C** lies on the other.
B satisfies $2x - 3y < 15$, so $\mathbf{P}_2 = \{\mathbf{C}\}$.

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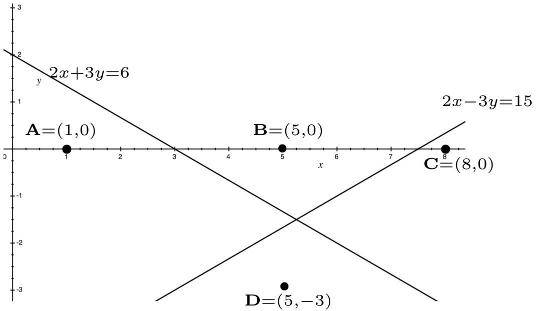


A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_1 = \{\mathbf{B}, \mathbf{C}\}$.

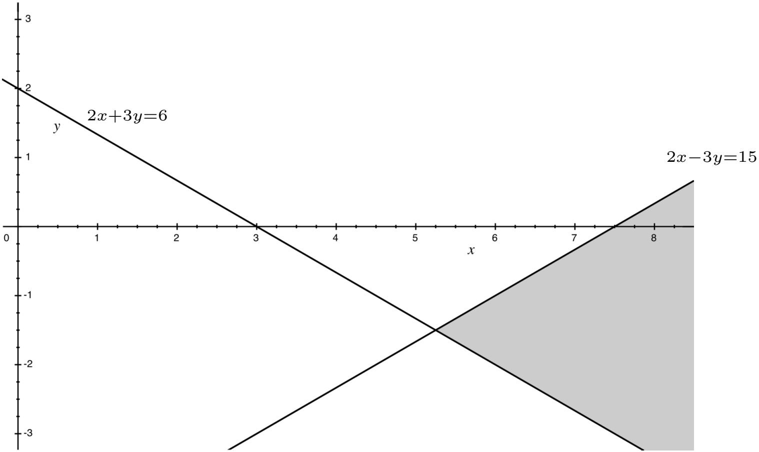
B lies on one side of $2x - 3y = 15$ and **C** lies on the other.

B satisfies $2x - 3y < 15$, so $\mathbf{P}_2 = \{\mathbf{C}\}$.

These are all the constraint lines so **C** is the feasible set.

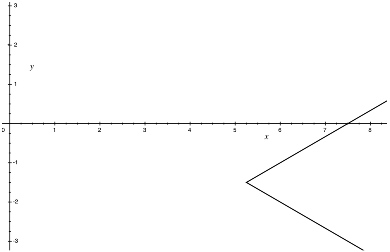
Determining the feasible set

The feasible set is shaded: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$



Determining the feasible set

Here is the boundary of the feasible set in the last example. It consists of two *rays* — parts of a line consisting of a point on the line and all points on the line lying to one side of that point.

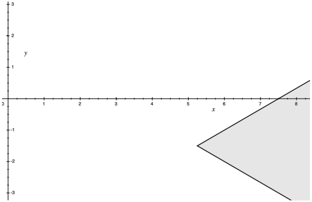


This would be the boundary of the feasible set for any of the four systems

$$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15 \qquad 2x + 3y \geqslant 6 \quad 2x - 3y > 15$$
$$2x + 3y > 6 \quad 2x - 3y \geqslant 15 \qquad 2x + 3y > 6 \quad 2x - 3y > 15$$

Determining the feasible set

The exact picture of the feasible set depends on whether the inequalities are $>$ or $\geq$



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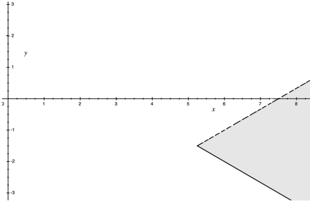

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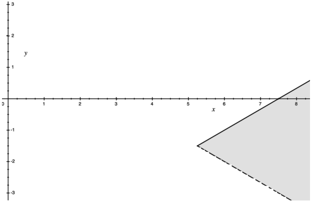
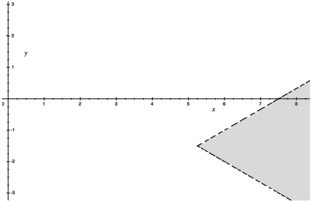


$2x + 3y > 6 \quad 2x - 3y \geqslant 15$



$2x + 3y > 6 \quad 2x - 3y > 15$

Determining the feasible set

Example Graph the feasible set for the system of inequalities:

$$x - y \geqslant 2 \qquad y + 2x \geqslant 6 \qquad y \geqslant 2$$

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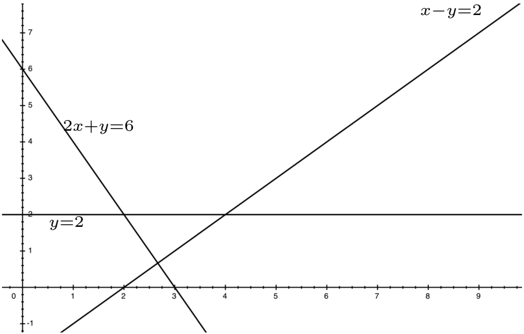
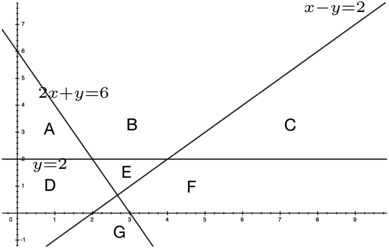


The three lines:

Warning: The x and y axes are NOT part of the system of constraint lines!

Determining the feasible set

There are 7 regions



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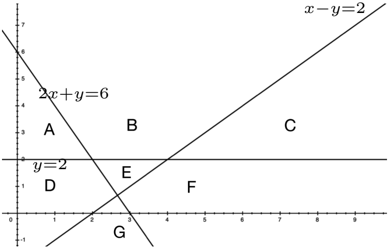

For $x - y \geqslant 2$ the "possible set" is $\mathbf{P}_1 = \{\mathrm{C}, \mathrm{F}, \mathrm{G}\}$ since $(4, 0)$ satisfies $x - y > 2$.

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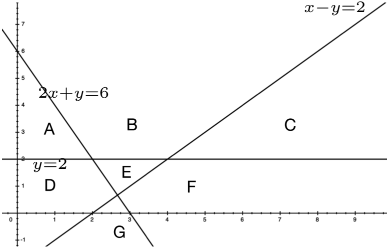

For $x - y \geqslant 2$ the "possible set" is $\mathbf{P}_1 = \{\mathrm{C}, \mathrm{F}, \mathrm{G}\}$ since $(4, 0)$ satisfies $x - y > 2$.

For $2x + y \geqslant 6$ is $\mathbf{P}_2 = \{\mathrm{C}, \mathrm{F}\}$ since $(4, 0)$ satisfies $2x + y > 2$.

Determining the feasible set

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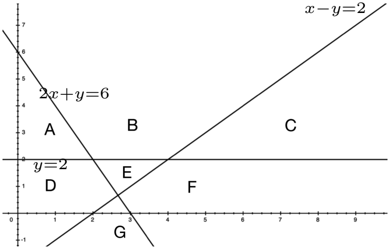

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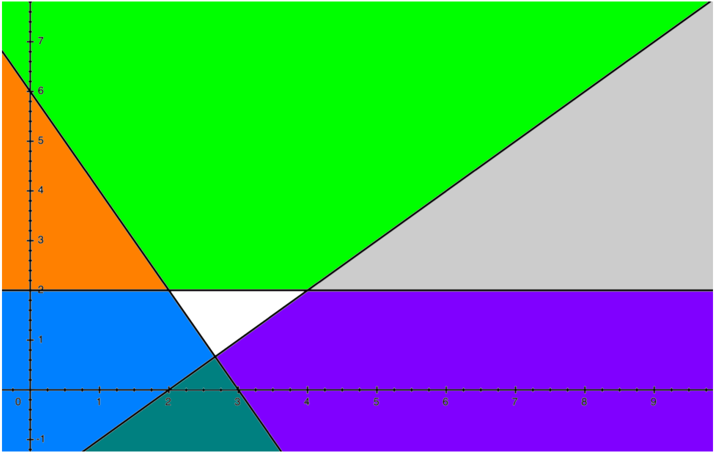
For $2x + y \geqslant 6$ is $\mathbf{P}_2 = \{\mathrm{C}, \mathrm{F}\}$ since $(4, 0)$ satisfies $2x + y > 2$.

Finally, if $y > 2$, $\mathbf{P}_3 = \{\mathrm{C}\}$ is all that is left and we have used all the lines.

Determining the feasible set

Here are the 7 regions that the constraint lines carve out.
The feasible set is gray.



Determining the feasible set

Why aren't there 8 regions?

Determining the feasible set

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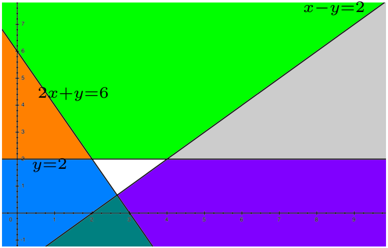
$x - y \leqslant 2$	$y + 2x \leqslant 6$	$y \leqslant 2$	$x - y \leqslant 2$	$y + 2x \leqslant 6$	$y \geqslant 2$
$x - y \leqslant 2$	$y + 2x \geqslant 6$	$y \leqslant 2$	$x - y \leqslant 2$	$y + 2x \geqslant 6$	$y \geqslant 2$
$x - y \geqslant 2$	$y + 2x \leqslant 6$	$y \leqslant 2$	$\mathbf{x - y \geqslant 2}$	$\mathbf{y + 2x \leqslant 6}$	$\mathbf{y \geqslant 2}$
$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \leqslant 2$	$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \geqslant 2$



The color of the constraints corresponds to the color in the diagram *except*:

- The black constraints yield the white region.
- The bold constraints yield an empty region.

Determining the feasible set

In general, if there are n constraint equations there will be at most 2^n regions, because there are 2^n ways to decide the $\leq$'s and $\geq$'s for each equation This follows of course from your general counting principles.

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In fact, it can be shown that if you draw n lines in the plane, you can create at most $(n^2 + n + 2)/2$ regions.

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In fact, it can be shown that if you draw n lines in the plane, you can create at most $(n^2 + n + 2)/2$ regions.

Typically you are only interested in one of the regions (the feasible set for your problem) and you can ignore the others.

Bounded and unbounded sets

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In the last example, the white triangle is bounded and the six other regions are unbounded.

The **corners** or **vertices** of the feasible set will be points at which constraint lines intersect. We will need to find the co-ordinates of the vertices of such a feasible set to solve the linear programming problems in the next section.

The intersection of a pair of lines

An easy way to find the intersection of a pair of lines (both non vertical), is to rearrange their equation to the (standard) form shown below and equate y values;

$$y = m_1 x + b_1 \text{ and } y = m_2 x + b_2$$

intersect where

$$m_1 x + b_1 = m_2 x + b_2.$$

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intersect where

$$m_1 x + b_1 = m_2 x + b_2.$$

If one of the lines is vertical its equation is $x = c$. Plug this value of x into the equation for the other line to find the y-value at the point of intersection

Example

Find the point of intersection of the lines:

$$2x + 3y = 6$$

$$2x - 3y = 15$$

Example

Find the point of intersection of the lines:

$$2x + 3y = 6$$

$$2x - 3y = 15$$

$$y = -\frac{2}{3}x + 2$$

$$y = \quad \frac{2}{3}x - 5$$

so $\frac{2}{3}x - 5 = -\frac{2}{3}x + 2$ or $\frac{4}{3}x = 2 + 5$. Then $4x = 3 \cdot (7) = 21$ so $x = \frac{21}{4}$. Then $y = \frac{2}{3}\left(\frac{21}{4}\right) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$. So $\left(\frac{21}{4}, -\frac{3}{2}\right)$ is the point of intersection.

Finding corners of a feasible set

To find the vertices/corners of the feasible set, graph the feasible set and identify which lines intersect at the corners.

Example Find the vertices of the feasible set corresponding to the system of inequalities:

$$2x + 3y \geqslant 6$$

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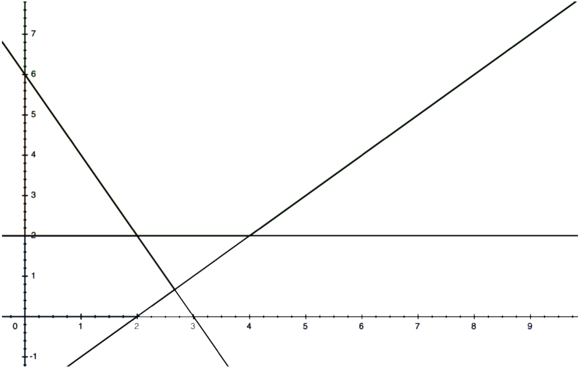
This is the same problem we just worked. The two lines are not parallel or equal so they intersect in one point, $\left(\frac{21}{4}, -\frac{3}{2}\right)$.

Finding corners of a feasible set

Example Find the vertices of the feasible set corresponding to the system of inequalities:

$$x - y \geqslant 2 \qquad y + 2x \geqslant 6 \qquad y \geqslant 2$$



Finding corners of a feasible set

No two of these three lines are parallel or equal so there are three vertices.

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$x - y = 2$ and $y + 2x = 6$ intersect as follows: $y = x - 2$, $y = -2x + 6$ so $x - 2 = -2x + 6$ or $3x = 6 + 2$ so $x = \frac{8}{3}$ and then $y = \frac{8}{3} - 2 = \frac{2}{3}$ so the intersection is $\left(\dfrac{8}{3}, \dfrac{2}{3}\right)$.

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$x - y = 2$ and $y = 2$ intersect as follows: $y = x - 2$, $y = 2$ so $x - 2 = 2$ or $x = 4$ and then $y = 2$ so the intersection is $(4, 2)$.

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$y = 2$ and $y + 2x = 6$ intersect as follows: $y = 2$, $y = -2x + 6$ so $2 = -2x + 6$ or $x = 2$ and then $y = 2$ so the intersection is $(2, 2)$.

There is only one vertex in the feasible set, $\boxed{(4, 2)}$.

Empty Feasible Sets

Sometimes there are no points in the feasible set for a system of inequalities.

Example Graph the feasible set for the system of inequalities:

$$x - y \geqslant 2 \quad x + y \leqslant 1 \quad y \geqslant 0 \quad x \geqslant 0$$

Empty Feasible Sets

The constraint lines, this time including both axes, divide the plane into 11 regions, so 5 of the potential regions must be empty.

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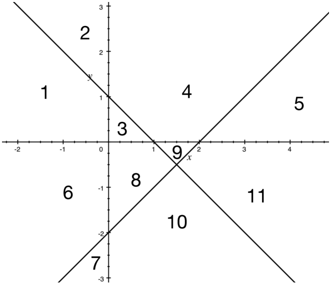

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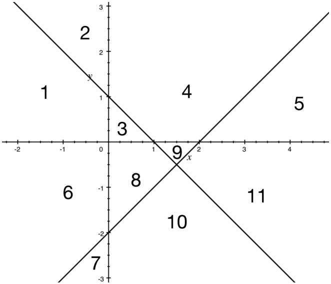

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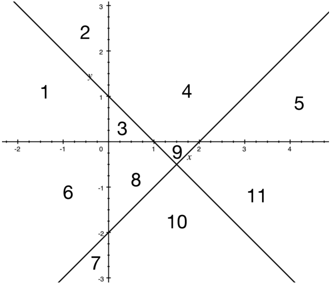

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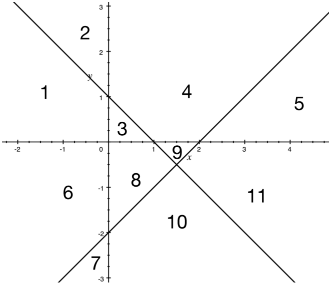

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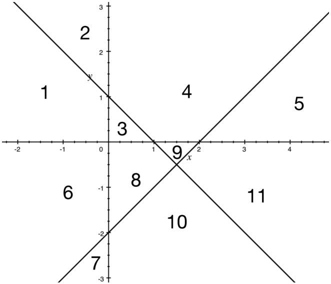

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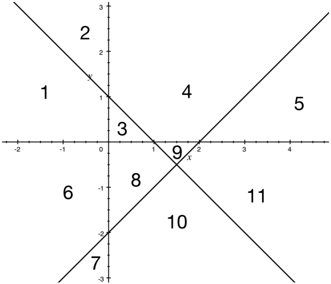

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No remaining region satisfies $y \geqslant 0$: $\mathbf{P}_3 = \emptyset$.

Warning: Do not make the mistake of stopping at $\mathbf{P}_2 = \{\mathbf{10}\}$ and claiming that is the feasible set. Do not stop until either $\mathbf{P}_n = \emptyset$ as here or until you have examined all the inequalities.

Setting up inequalities for a problem

Example Mr. Carter eats a mix of Cereal A and Cereal B for breakfast. The amount of calories and sodium per 25g for each is shown in the table below. Mr. Carter's breakfast should provide at least 480 calories but less than 700 milligrams of sodium.

	Cereal A	**Cereal B**
Calories(per 25g)	100	140
Sodium(mg per 25g)	150	190

Let x denote the number of 25g units of Cereal A that Mr. Carter has for breakfast and let y denote the number of 25g unit of Cereal B he has. What are the constraints on the amounts of each cereal?

Setting up inequalities for a problem

$$
\begin{aligned}
100x + 140y &\geqslant 480 \qquad \textit{Calories} \\
150x + 190y &< 700 \qquad \textit{Sodium} \\
x \geqslant 0 \quad y &\geqslant 0 \qquad non-negative\ conditions
\end{aligned}
$$

Setting up inequalities for a problem

Example A juice stand sells two types of fresh juice in 12 oz cups, the Refresher and the Super-Duper. The Refresher is made from 3 oranges, 2 apples and a slice of ginger. The Super-Duper is made from one slice of watermelon, 3 apples and one orange. The owners of the juice stand have 50 oranges, 40 apples, 10 slices of watermelon and 15 slices of ginger. Let x denote the number of Refreshers they make and let y denote the number of Super-Dupers they make. What is the set of constraints on x and y?

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$$2x + 3y \leqslant 40 \quad Apples \qquad 3x + 1y \leqslant 50 \quad Oranges$$

$$0x + 1y \leqslant 10 \quad Watermelon \qquad 1x + 0y \leqslant 15 \quad Ginger$$

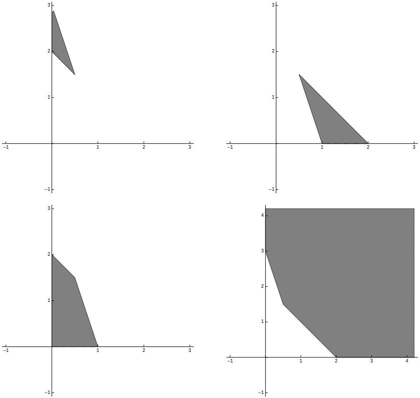
$$x \geqslant 0 \quad y \geqslant 0 \qquad non-negative\ conditions$$

Old exam question I

Select the graph of the feasible set of the system of linear inequalities given by:

$$\begin{array}{rcrcrcrcrcr} x & & & \geqslant & 0 & & & y & \geqslant & 0 \\ 3x & + & y & \leqslant & 3 & 2x & + & 2y & \leqslant & 4 \end{array}$$

where the shaded area is the feasible set.



Old exam question I

A quick solution is to note that $(0, 0)$ satisfies all the inequalities. Hence the first graph on line 2 is the only possible answer.

Or just draw the lines and shade the feasible set.

Old exam question I

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

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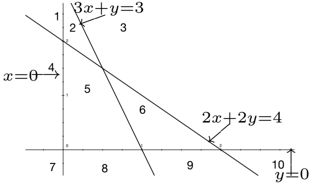
The lines and regions are



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$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

$(0,0)$ satisfies $3x + y \leqslant 3$:

$P_0 = \{1, 2, 4, 5, 7, 8\}$

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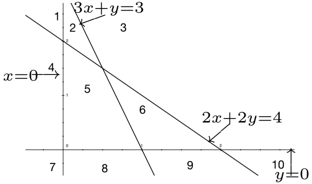

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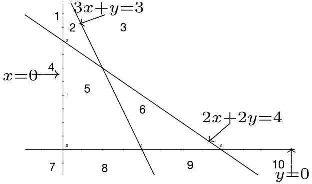

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$\mathrm{P}_0 = \{1, 2, 4, 5, 7, 8\}$

$(0, 0)$ satisfies $2x + 2y \leqslant 4$:

$\mathrm{P}_1 = \{4, 5, 7, 8\}$

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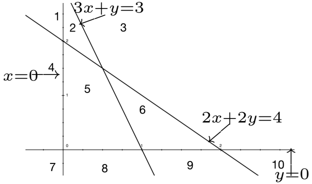

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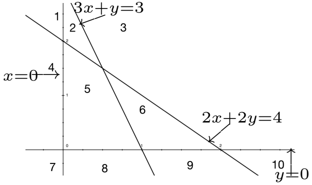

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$(1, 0)$ satisfies $x \geqslant 0$:
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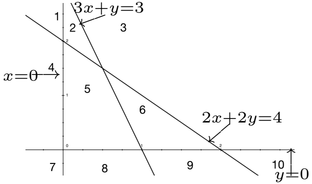

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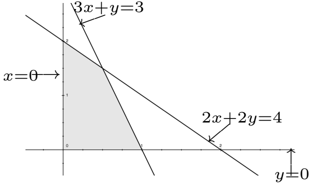

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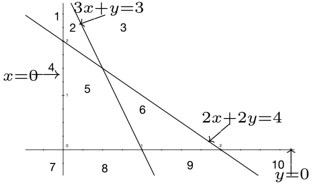

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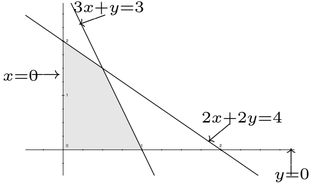

Note that the feasible set is bounded.

Old exam question II

A student spending spring break in Ireland wants to visit Galway and Cork. The student has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros and each day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork. Which of the following sets of constraints describe the constraints on the student's time and money for the visits?

Old exam question II

Recall: The student has at most 7 days available, at most 500 euros to spend. A day spent in Galway will cost 50 euros and a day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork.

$$(a)\quad \begin{aligned} x + y &\leqslant 7 \\ 50x + 60y &\leqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(b)\quad \begin{aligned} x + 7y &\leqslant 500 \\ 50x + 60y &\leqslant 1000 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(c)\quad \begin{aligned} x + y &\leqslant 7 \\ 60x + 50y &\leqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(d)\quad \begin{aligned} x + y &\geqslant 7 \\ 50x + 60y &\geqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(e)\quad \begin{aligned} x + y &\geqslant 7 \\ 60x + 50y &\geqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

Old exam question II

$$\begin{array}{rcrcrl} x & + & y & \leqslant & 7 & \text{Days} \\ 50x & + & 60y & \leqslant & 500 & \text{euros} \\ x \geqslant 0 & & y \geqslant 0 & & & \end{array}$$