

Exam 2

March 8, 2018

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May the odds be ever in your favor!

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Multiple Choice

1. (5 pts.) Suppose an experiment has sample space $S = \{a, b, c\}$ with $P(\{a, b\}) = 0.8$ and $P(\{b, c\}) = 0.55$. Find $P(\{a, c\})$.

(a) 0.30

(b) 0.65

(c) 0.20

(d) 0.35

(e) 0.45

2. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A five card hand is dealt at random. What is the probability of getting (exactly) three clubs or (exactly) three spades?

(a) $\frac{C(13, 3) \cdot C(49, 2)}{C(52, 5)}$

(b) $\frac{2 \cdot C(13, 3)}{C(52, 5)}$

(c) $\frac{2 \cdot C(13, 3) \cdot C(39, 2)}{C(52, 5)}$

(d) $\frac{2 \cdot C(13, 3)}{C(52, 5)}$

(e) $\frac{C(13, 3) \cdot C(39, 2)}{C(52, 5)}$

3. (5 pts.) I toss a fair six sided die three times. What is the probability that I see at least one even number and at least one odd number?

(a) $\frac{3}{4}$

(b) $\frac{7}{8}$

(c) $\frac{1}{8}$

(d) $\frac{1}{4}$

(e) $\frac{1}{2}$

4. (5 pts.) Suppose two cards are dealt at random from a standard 52 card deck. What is the probability that the first one was a heart if it is known that the second one was a heart? [Hint: tree diagram.]

(a) $\frac{13}{17}$

(b) $\frac{1}{17}$

(c) $\frac{1}{4}$

(d) $\frac{4}{17}$

(e) $\frac{13}{51}$

5. (5 pts.) The 50 students taking MATH 10120 last year were polled about their music preferences. 35 said they like Justin Bieber, 25 said they like Beyoncé and 15 said they like both. If a student is selected at random and it is known that they like Beyoncé, what is the probability that they also like Justin Bieber?

- (a) $\frac{3}{5}$ (b) $\frac{1}{10}$ (c) $\frac{3}{7}$
(d) $\frac{3}{10}$ (e) $\frac{7}{10}$

6. (5 pts.) Let E and F be two events of an experiment. Which of the following statements is FALSE?

- (a) if E and F are independent, then $P(F|E) = P(F)$
(b) if E and F are independent, then $P(E \cap F) = P(E)P(F)$
(c) if E and F are mutually exclusive, then $P(E \cup F) = P(E)P(F)$
(d) if E and F are mutually exclusive, then $E \cap F = \emptyset$
(e) if E and F are independent, then $P(E|F) = P(E)$

7. (5 pts.) I have five envelopes in front of me; two of them have some money in them, and the other three are empty. I open the envelopes one after another, in random order. I stop as soon as I have found an envelope that has money in it. What is the probability that I stop just after I open the third envelope? (I.e., that I find money for the first time in the third envelope I try?) [Hint: tree diagram.]

(a) $\frac{3}{10}$

(b) $\frac{13}{20}$

(c) $\frac{1}{5}$

(d) $\frac{1}{10}$

(e) $\frac{2}{5}$

8. (5 pts.) My computer has two fans, that operate independently. The probability that the first is still working after 24 hours of continuous use is 0.78, and the probability that the second is still working after 24 hours of continuous use is 0.67. What is the probability that **exactly one of them** is still working after 24 hours of continuous use?

(a) 0.1474

(b) 0.5226

(c) 0.2574

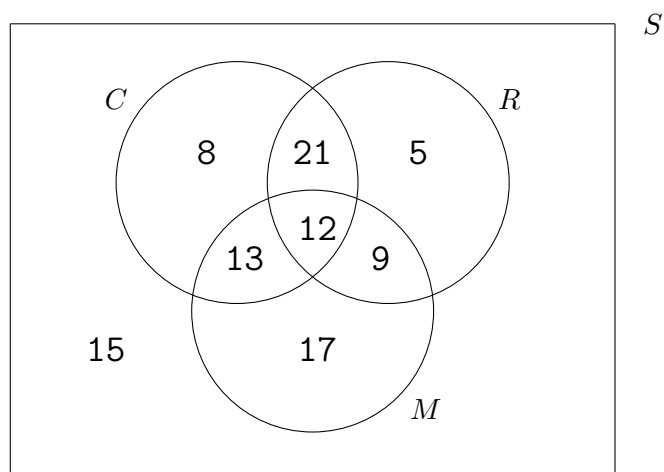
(d) 0.4048

(e) 0.4774

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The 100 students at St. Patrick's College have the option to join any of three clubs: climbing club, running club and math club (they could join any number, or none). Let C , R and M denote the events that a randomly chosen student is in the climbing, running or math club, respectively. The following Venn diagram collects the relevant information.



For all of the following, present your final answers as a fraction.

- Find the probability that a randomly picked student is part of the climbing club.
- If a randomly picked student is part of the running club, what is the probability that they are also either in the climbing club or the math club (or both)?
- If a randomly picked student is not part of the running club, what is the probability that they are in the math club?

12. (10 pts.) Justin plays a carnival game that works as follows: First he tosses a fair coin. Next he gets to pick marbles (without replacement) from a bag containing 5 red and 3 white marbles. If the coin comes up heads, he picks one marble. If it comes up tails, he picks two marbles. Justin wins the game if he picks at least one white marble.

For all of the following, present your final answers either as a fraction or as a decimal.

(a) Draw a complete tree diagram for the above experiment. All branches of the diagram should be labeled with probabilities and events.

(b) What is the probability of winning the game?

(c) Given that Justin won the game, what is the probability that the coin came up heads?

13. (10 pts.) An experiment consists of tossing a fair coin 8 times in a row. For this question, you can leave your answers as expressions involving combination numbers, factorials, powers, etc.

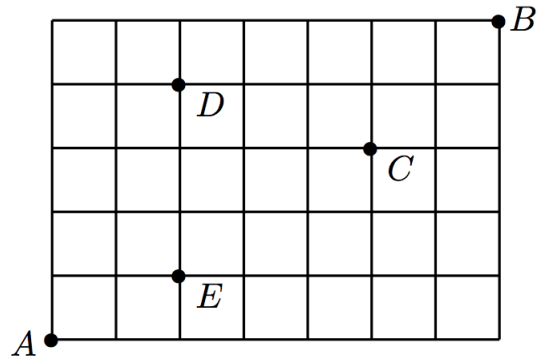
(a) What is the probability of the following sequence of tosses: heads, heads, tails, heads, tails, tails, heads, heads?

(b) What is the probability of getting at most three heads?

(c) What is the probability getting exactly four heads, which all occur consecutively?

(d) Are the events “getting at most three heads” and “getting at most three tails” independent? Explain.

14. (10 pts.) John travels from A to B along the city grid shown below, using as few blocks as possible, choosing his particular route randomly (all routes equally likely). For this question, answers may be left in terms of combination numbers, factorials, etc.



- (a) What's the probability that along the way he passes C ?
- (b) At some point he is seen passing E . Now what's the probability that along the way he passes C ?
- (c) At some point he is seen passing D . Now what's the probability that along the way he passes C ?

15. (10 pts.) Professor Bunsen always starts her Alchemy 101 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that she teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When she tries to turn lead into gold, the result always ends with a explosion; when she brews the elixir of life, there is a 50% chance of an explosion, and when she creates the Philosopher's stone, 8 times out of 10 there is an explosion.

(a) The next time Professor Bunsen teaches the course, what is the probability of an explosion happening?

(b) What is the probability that either there is an explosion, or the professor attempts to brew the elixir of life?

(c) Fr. Jenkins wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what (should he conclude) is the probability that he has just missed a demonstration of brewing the elixir of life?

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