Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name: SOLUTIONS

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# Exam 2 Practice (actual exam from Fall 2017)

**February 28, 2018**

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

**The partial credit problems should be answered on the page where the problem is given.** The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an × through your answer to each problem.

| | | | | | |
|---|---|---|---|---|---|
| 1. | ~~(a)~~ | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | ~~(c)~~ | (d) | (e) |
| 3. | (a) | ~~(b)~~ | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | ~~(e)~~ |
| 5. | (a) | (b) | (c) | ~~(d)~~ | (e) |
| 6. | (a) | (b) | (c) | ~~(d)~~ | (e) |
| 7. | (a) | ~~(b)~~ | (c) | (d) | (e) |
| 8. | ~~(a)~~ | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | ~~(e)~~ |
| 10. | ~~(a)~~ | (b) | (c) | (d) | (e) |

| | |
|---|---|
| MC. | ______ |
| 11. | ______ |
| 12. | ______ |
| 13. | ______ |
| 14. | ______ |
| 15. | ______ |
| Tot. | ______ |

## Multiple Choice

**1.** (5 pts.) Suppose an experiment has sample space $S = \{a, b, c\}$ with $P(\{a\}) = 0.4$, $P(\{a, c\}) = 0.65$ and $P(\{b, c\}) = 0.6$. Find $P(\{a, b\})$.

~~(a)~~ 0.75 (b) 0.6 (c) 0.25 (d) 0.35 (e) 0.2

$P(a) = 0.4$

$P(c) = P(\{a,c\}) - P(a) = 0.65 - 0.4 = 0.25$

$P(b) = P(\{b,c\}) - P(c) = 0.6 - 0.25 = 0.35$

$P(\{a,b\}) = 0.4 + 0.35 = 0.75$

**2.** (5 pts.) We toss a fair six sided die twice and note down the numbers that show up. What is the probability that either a 5 or a 6 or both appear?

(a) $\frac{1}{4}$ (b) $\frac{11}{36}$ ~~(c)~~ $\frac{5}{9}$ (d) $\frac{1}{3}$ (e) $\frac{5}{6}$

$\frac{20}{36} = \frac{5}{9}$

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | x | x |
| 2 | | | | | x | x |
| 3 | | | | | x | x |
| 4 | | | | | x | x |
| 5 | x | x | x | x | x | x |
| 6 | x | x | x | x | x | x |

**3.** (5 pts.) I pick a random card from a standard 52 card deck. What is the probability that the card I pick is red or an ace?

(a) $\frac{1}{2}$ ~~(b)~~ $\frac{7}{13}$ (c) $\frac{29}{52}$ (d) $\frac{1}{13}$ (e) $\frac{841}{2704}$

$P(\text{Red}) = 26/52$

$P(\text{Ace}) = \frac{4}{52}$

$P(\text{Red} \cap \text{Ace}) = 2/52$

$$P(\text{Red} \cup \text{Ace}) = P(\text{Red}) + P(\text{Ace}) - P(\text{Red} \cap \text{Ace})$$
$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

**4.** (5 pts.) A coin is biased so that the probability that it comes up heads is two thirds. If the coin is tossed eight times, what is the probability that it results in exactly 4 heads?

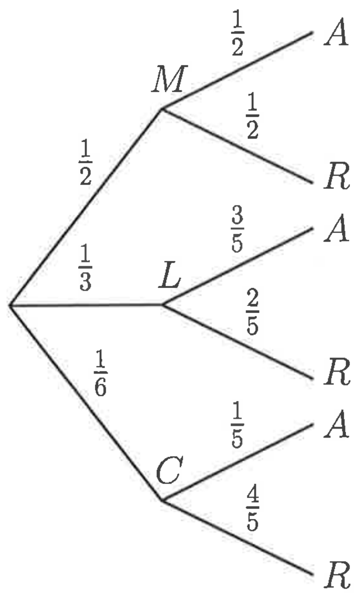
(a) $C(8,4) \cdot \frac{1}{2^8}$ (b) $C(8,4)^2 \cdot \frac{1}{2^8}$ (c) $\left(\frac{2}{3}\right)^4$

(d) $\left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$ ~~(e)~~ $C(8,4) \cdot \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$

Once we choose which tosses are heads/tails, the probability is $\left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^4$

there are $\binom{8}{4}$ ways to choose which 4 out of 8 are heads.

**5.** (5 pts.) Moe (M), Larry (L) and Curly (C) are admissions officers at Stooge University. For each student who applies, they either accept (A) that student or reject (R) that student. For a randomly selected applicant, the following tree diagram gives the probabilities that Moe, Larry or Curly handled their application, and it gives the conditional probabilities that the applicant was accepted or rejected by the different officers.




$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{3} + \frac{2}{5} = \frac{2}{15}$

$\frac{1}{6} \times \frac{4}{5} = \frac{2}{15}$

Find the probability that a randomly chosen applicant was rejected.

(a) $\frac{17}{30}$ (b) $\frac{17}{20}$ (c) $\frac{17}{10}$ (d) $\frac{31}{60}$ (e) $\frac{29}{60}$

$\frac{1}{4} + \frac{2}{15} + \frac{2}{15} = \frac{31}{60}$

**6.** (5 pts.) A partial deck of cards contains 10 clubs, 8 diamonds, 6 hearts and 4 spades. Two cards are chosen at random without replacement. If the first is the king of diamonds, what is the probability that the second is red?

(a) $\frac{14}{28}$ (b) $\frac{13}{28}$ (c) $\frac{1}{13}$ ~~(d)~~ $\frac{13}{27}$ (e) $\frac{1}{14}$

Once diamond is removed, $7+6=13$ red cards left out of 27 total

$\rightarrow 13/27$

**7.** (5 pts.) Which of the following does *not* imply that events $E$ and $F$ are independent?

(a) $P(E|F) = P(E)$

(b) $P(E \cup F) = P(E) + P(F)$

(c) $P(F|E) = P(F)$

(d) $P(E \cap F) = P(E)P(F)$

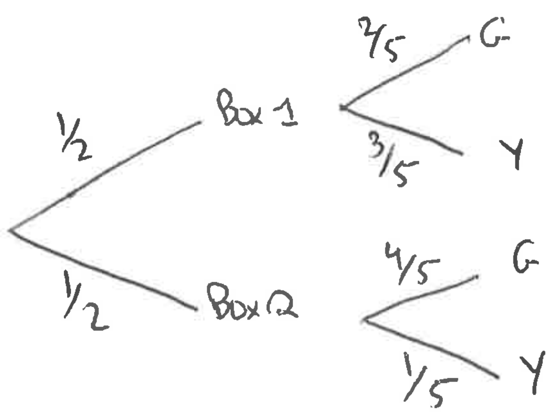
(e) $P(E \cup F) = P(E) + P(F) - P(E)P(F)$

Recall a, ~~b~~, c, d are conditions to check independence

e is inclusion-exclusion for indept events.

**8.** (5 pts.) Box I contains 2 green balls and 3 yellow balls. Box II contains 4 green balls and 1 yellow ball. A ball is chosen from one of the boxes at random. If the ball is green, what is the probability that it came from Box II? (Hint: draw a tree diagram.)

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$ (e) $\frac{3}{10}$




$$P(\text{Box } 2 \mid G) = \frac{P(\text{Box } 2 \cap G)}{P(G)}$$

$$= \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{2}{5}} = \frac{2/5}{2/5 + 1/5} = \frac{2}{3}$$

**9.** (5 pts.) Bob has three standard decks of cards. He draws one card at random from each deck (independently). What is the probability that he gets exactly two kings among those three cards?

(a) $\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)$ (b) $3\left(\frac{1}{13}\right)^2$ (c) $\left(\frac{1}{13}\right)^2$

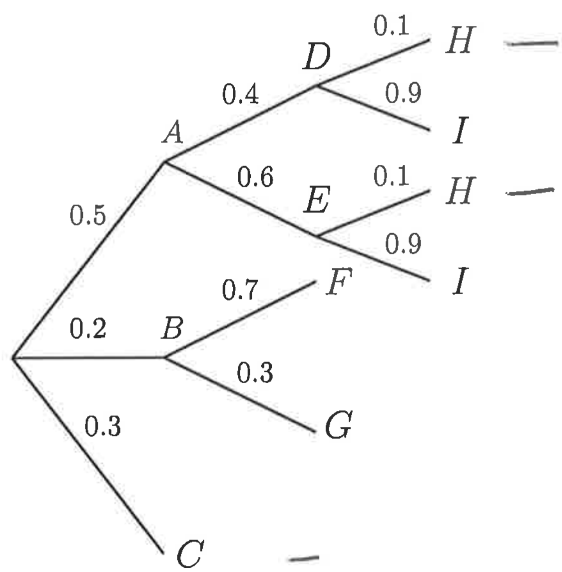
(d) $\frac{2}{3}$ (e) $3\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)$

Similar to #4. ~~(3 way~~ $\binom{3}{2}$ ways to choose which cards ~~is~~ are K.

For each, the probability is $\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)$ one is not king

two are kings.

**10.** (5 pts.) Some game winds up with the following as its tree diagram.




The numbers represent probabilities and it is not relevant to this problem what $A, B, C, D, E, F, G, H, I$ stand for. The player is done as soon as they reach one of the endpoints $C$, $F$, $G$, $H$ or $I$ (and note that $H$ and $I$ appear twice). The player wins if they reach either $H$ or $C$. Assuming that the player won, what is the probability that they ended up at $C$ (to three decimal places)?

(a) 0.857 (b) 0.300 (c) 0.350 (d) 0.050 (e) 0.800

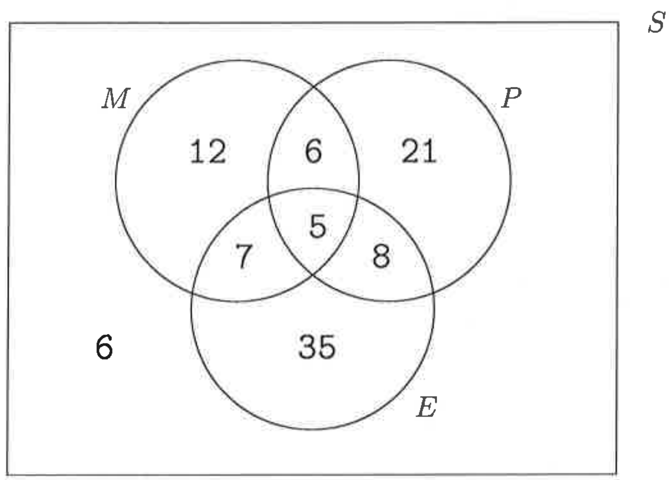
$$P(C|W) = \frac{P(C\cap W)}{P(W)} = \frac{0.3}{(0.5\times 0.4\times 0.1)+(0.5\times 0.6\times 0.1)+0.3}$$

$\approx 0.857$

## Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) A club at a certain university has 100 members. Some are math majors, some are political science majors, some are education majors and some are none of these. Let $M$, $P$ and $E$ denote the events that a randomly chosen student is a math, political science or education major, respectively. The following Venn diagram collects the relevant information.




$n(S) = 100$

Find the following probabilities. Please leave your answers as **fractions**. Note that for a set $A$, the notation $A'$ means the same thing as the notation $A^c$, the complement of $A$.

(i) $P(M) = \dfrac{12+6+5+7}{100} = 30/100$

(ii) $P(E \cap P \cap M') = \dfrac{8}{100}$

(iii) $P(E \cap P) = \dfrac{5+8}{100} = \dfrac{13}{100}$

(iv) $P(E'|M) = \dfrac{n(E' \cap M)}{n(M)} = \dfrac{12+6}{30 \leftarrow \text{from (a)}} = 18/30 = 3/5$

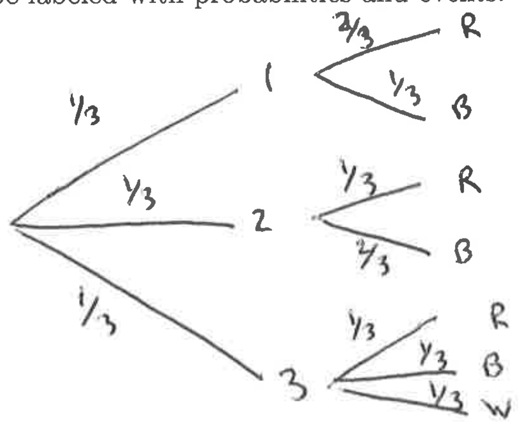
(v) $P(E' \cup M) = \dfrac{12+6+7+5+21+6}{100} = \dfrac{57}{100}$

**12.** (10 pts.) I have a bag with one red and one blue ball. Consider the following experiment: I toss a fair three sided die (three sided dice do exist!) and do one of the following:

- If the die shows a 1, I add one red ball to the bag.
- If the die shows a 2, I add one blue ball to the bag.
- If the die shows a 3, I add one white ball to the bag.

After adding a ball to the bag, I pick a random ball from the bag and note its color.

(a) Draw a complete tree diagram for the above experiment. All branches of the diagram should be labeled with probabilities and events.




(b) Let $R$ be the event that I drew a red ball from the bag. Compute $P(R)$.

$$P(R) = \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$

$$= 4/9$$

(c) Given that I drew a red ball from the bag, what is the probability that the die showed a 1?

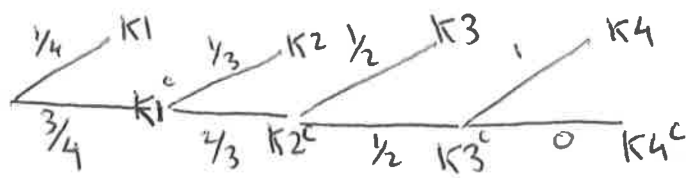
$$P(1 \mid R) = \frac{P(1 \cap R)}{P(R)} = \frac{1/3 \times 2/3}{4/9} = \frac{2/9}{4/9} = \frac{1}{2}$$

from (b)

**13.** (10 pts.) Jake has four different keys on his keychain, only one of which opens his front door.

(a) Suppose Jake forgets which key opens his front door and tries each of the keys, one after another, until he is able to open the door. (Note that once he tries a key and it doesn't work he doesn't try using the same key again. He stops when the door is open.)

Draw a complete tree diagram for the above experiment. All branches of the diagram should be labeled with probabilities and events. (Use $K1$ for the event that the first key he tries opens the door, $K2$ for the event that the second key he tries opens the door, et cetera).




(b) Find the probability that the fourth key he tries is the correct one.

$$\left(\frac{\cancel{3}}{4}\right)\left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{1}{\cancel{2}}\right)(1) = 1/4$$

(c) Suppose now that he gets back home drunk. Unfortunately, now he is unable to remember which keys he tried before. So it is possible that he tries a key he has tried before. He tries to open the door 5 times before giving up. What is the probability that he is unable to open the door after 5 tries? (Hint: Now each try is independent of the others!)

Now the tries are independent.

P(first fails) P(second fails) P(3rd fails) P(4th fails) P(5th fails)

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^5 \approx 0.237$$

**14.** (10 pts.) Suppose we have a sample space $S$ with $n(S) = 100$. Let $A, B, C$ be events in the sample space, with

- $n(A) = 20$,
- $n(B) = 25$,
- $n(C) = 60$,
- $n(A \cap B) = 0$,
- $n(A \cap C) = 5$,
- $n(B \cap C) = 15$.

(Hint: A Venn Diagram might help with this problem.)

(a) Find $P(A \cap C)$.

$$= \frac{n(A \cap C)}{n(S)} = \frac{5}{100} = 0.05$$

(b) Are the events $A$ and $C$ dependent or independent? Explain.

$P(A \cap C) = 0.05$ $\quad P(A) = \frac{20}{100}$, $P(C) = \frac{60}{100}$

$P(A)P(C) = \frac{20}{100} \times \frac{60}{100} = 0.12 \neq 0.05 \rightarrow A, C$ dependent

(c) Find $P(B|C)$.

$$= \frac{P(B \cap C)}{P(C)} = \frac{n(B \cap C)}{n(C)} = \frac{15}{60} = 0.25$$

(d) Are the events $B$ and $C$ dependent or independent? Explain.

$P(B) = 0.25$, $P(B|C) = 0.25$

$\rightarrow B, C$ indept

(e) Are the events $A$ and $B$ dependent or independent? Explain.

$P(A \cap B) = 0$

But $P(A) \neq 0$ and $P(B) \neq 0$ (A, B are mutually exclusive)

so A, B dependent.

**15.** (10 pts.) A bag contains 4 red balls and 6 blue ones. You pick out three of them sequentially *without replacement*. In this problem you may leave your answer in terms of combinations, permutations, powers and fractions.

(i) What is the probability that all three are the same color?

$$\frac{C(4,3)+C(6,3)}{C(10,3)}$$

(ii) What is the probability that the first is red, the second is blue and the third is red?

$$\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}$$

(iii) Suppose that you pick the three balls *with* replacement. Now what is the probability that all three are the same color?

$$\left(\frac{4}{10}\right)^3+\left(\frac{6}{10}\right)^3$$