Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name: SOLUTIONS

Instructors: Basit/Galvin

Exam 3

April 19, 2018.

This exam is in two parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, etc.. Write your name on the title page and put your initials at the top of every page.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	~~(a)~~	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	~~(e)~~
3.	(a)	(b)	~~(c)~~	(d)	(e)
4.	(a)	(b)	(c)	(d)	~~(e)~~
5.	(a)	(b)	(c)	(d)	~~(e)~~
6.	(a)	(b)	(c)	~~(d)~~	(e)
7.	(a)	(b)	(c)	~~(d)~~	(e)
8.	(a)	(b)	~~(c)~~	(d)	(e)
9.	(a)	~~(b)~~	(c)	(d)	(e)
10.	(a)	(b)	(c)	~~(d)~~	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Multiple Choice

1. (5 pts.) I played eight holes of golf at the Warren Golf Course, and here were my scores:

12, 6, 6, 6, 10, 8, 8, 16.

What are my mean, mode and median scores?

(a) ~~X~~ mean 9, mode 6, median 8 (b) mean 9, mode 6, median 9

(c) mean 10.4, mode 6, median 8 (d) mean 10.4, mode 8, median 7

(e) mean 9, mode 8, median 8

6 6 6 8 8 10 12 16

mean: $\frac{6\times3+8\times2+10+12+16}{8} = \frac{18+16+10+12+16}{8} = 72/8 = 9$

mode = 6 (6 appears 3 times)

median = $\frac{8+8}{2} = 8$ (average of middle 2 numbers)

2. (5 pts.) I sample six chocolate chip cookies from SDH and find that they have the following numbers of chocolate chips:

21, 20, 21, 22, 24, 24.

The sample mean of the number of chocolate chips is thus 22 (trust me on this). What is the sample standard deviation of the number of chocolate chips? (Note: I'm asking for *standard deviation*, not variance, and for the *sample* standard deviation, not population standard deviation. Answers rounded to two decimal places.)

(a) 2.8 (b) 1 (c) 1.87 (d) 3.5 (e) ~~X~~ 1.67

x	(x-M)	$(x-M)^2$
21	-1	1
20	-2	4
21	-1	1
22	0	0
24	2	4
24	2	+ 4
		14

Sample variance = $14/(6-1) = 14/5 = 2.8$

sample std dev = $\sqrt{2.8} \approx 1.67$

3. (5 pts.) The table below shows the probability distribution of a random variable X. What is the probability that X takes a value between 5 and 15, inclusive (i.e., what is $P(5 \le X \le 15)$)?

Possible value k	0	5	10	15	20
$P(X = k)$	.12	.15	?	.24	.21

(a) .28 (b) .72 (c) .67 (d) .33 (e) .39

$P(5 \le X \le 15) = P(5) + P(10) + P(15)$
$= 1 - (P(0) + P(20)) = 1 - (0.12 + 0.21) = 1 - 0.33$
$= 0.67$

OR $P(10) = 1 - 0.12 - 0.15 - 0.24 - 0.21$
$= 0.28$

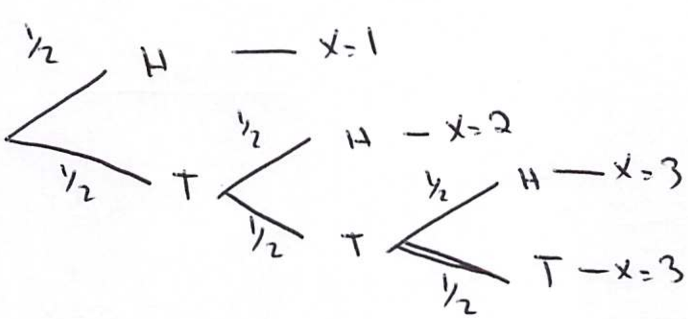
$\rightarrow P(5 \le X \le 15) = 0.15 + 0.28 + 0.24$
$= 0.67$

4. (5 pts.) I toss a fair coin. If the coin comes up heads, I stop. If it comes up tails, I toss the coin again. If it comes up heads the second time, I stop. If it comes up tails the second time, I toss the coin one more time, and then stop.

Let X be the number of times I toss the coin during this experiment. What is the expected value of X?

(a) 2.25 (b) 1.875 (c) 2

(d) 1.5 (e) 1.75



k	$P(X=k)$
1	1/2
2	1/4
3	$1/8 + \frac{1}{8} = 1/4$

$E[X] = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{4}$
$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$
$= 1.75$

5. (5 pts.) A subcommittee of 2 people is to be selected from a committee of 4 people, all selections equally likely. The full committee has two men and two women. Let X be the number of women on the subcommittee. Which of the following is the probability distribution table of X?

(a)

k	$P(X = k)$
0	1/8
1	3/8
2	3/8

(b)

k	$P(X = k)$
0	1/4
1	1/2
2	1/4

(c)

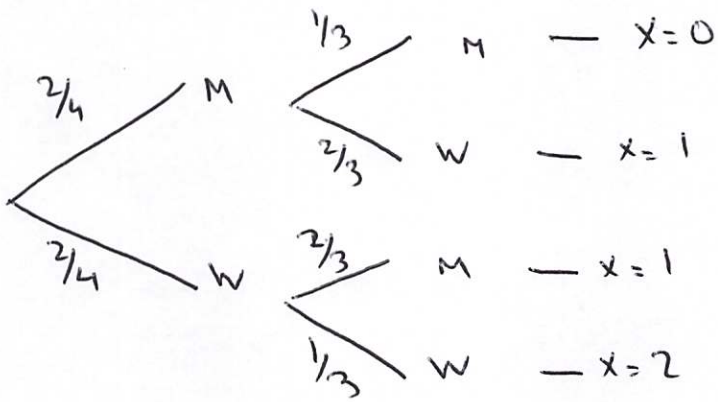
k	$P(X = k)$
0	1/3
1	1/3
2	1/3

(d)

k	$P(X = k)$
0	1/6
1	1/3
2	1/2

~~(e)~~

k	$P(X = k)$
0	1/6
1	2/3
2	1/6



k	P(X=k)
0	1/6
1	2/3
2	1/6

6. (5 pts.) In an archery contest, Alice gets to shoot seven arrows at the target. Suppose that the probability that she hits the target on any particular try is 0.7, independent of other tries. What is the probability that at least six of her arrows hit the target?

(a) 0.118 (b) 0.247 (c) 0.671

~~(d)~~ 0.329 (e) 0.035

X = # of arrows hitting target

→ X is Binomial with $n = 7$, $p = 0.7$

$$P(X \geq 6) = P(X=6) + P(X=7)$$
$$= \binom{7}{6}(0.7)^6(0.3) + \binom{7}{7}(0.7)^7(0.3)^0$$
$$\approx 0.329$$

7. (5 pts.) In an experiment, you roll a fair six sided die ten times. Let X be the number of times a 5 or 6 shows up. What is the expected value of the random variable X?

(a) $\frac{10}{9}$ (b) $\frac{5}{3}$ (c) $\frac{20}{3}$

(d) $\frac{10}{3}$ (e) $\frac{20}{9}$

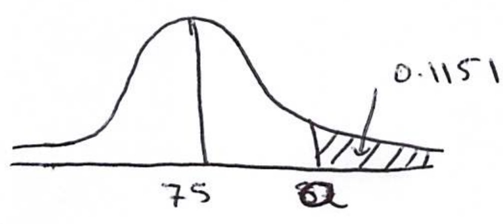
X is Binomial with $n = 10$, $p = 1/3$

$\to E[X] = np = 10/3$

8. (5 pts.) Let X be a random variable with the normal distribution with mean $\mu = 75$ and standard deviation $\sigma = 10$. If you know that $P(X \geq a) = 0.1151$, what is the value of a?

(a) 63 (b) 65 (c) 87

(d) 73 (e) 67



$z = \frac{a-75}{10}$

~~Area~~ $P(Z \geq z) = 0.5 - A(a) = 0.1151$

$A(z) = 0.3849$

$\to z = +1.2$

$\to 1.2 = \frac{a-75}{10} \to a = 87$

9. (5 pts.) Which of the following points satisfies all the inequalities below?

$$\begin{aligned} 3x - 2y &\leq 0 \\ 2x + y &\geq 8 \\ x &\leq 6 \\ y &\geq 0 \end{aligned}$$

(a) $(2,1)$ ~~(b)~~ $(2,6)$ (c) $(3,-1)$ X

(d) $(5,6)$ (e) $(7,4)$ X

Just plug in and check (c and e are easy to see)

10. (5 pts.) Fred and George plan to launch two new products: Extendable Ears and Skiving Snackboxes at their joke shop, Weasleys' Wizard Wheezes. For budgetary reasons they can spend at most 300 galleons on production. It costs 5 galleons to produce each set of Extendable Ears and 7 galleons to produce each of the Skiving Snackboxes. Before they open up shop, they want to have at least 25 items in stock.

Find the inequalities relating the number of Extendable Ears produced (x) and the number of Skiving Snackboxes produced (y). (Warning: pay close attention to the direction of the inequalities when you choose your answer, and pay attention to "at most" versus "at least" in the statement of the problem.)

(a) $\boxed{\begin{aligned} 5x + 7y &\geq 25 \\ x + y &\leq 300 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}}$ (b) $\boxed{\begin{aligned} x + y &\leq 25 \\ 5x + 7y &\leq 300 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}}$ (c) $\boxed{\begin{aligned} x + y &\leq 25 \\ 5x + 7y &\geq 300 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}}$

~~(d)~~ $\boxed{\begin{aligned} x + y &\geq 25 \\ 5x + 7y &\leq 300 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}}$ (e) $\boxed{\begin{aligned} x + y &\geq 25 \\ 5x + 7y &\leq 300 \end{aligned}}$

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (12 pts.) I roll an fair four-sided die twice, and I record the bigger of the two numbers that come up on the rolls. (So, for example, if I roll a 2 followed by a 4, I record "4"; if I roll a 3 followed by a 3, I record "3"). Let X be the number that I record; so the possible values of X are 1, 2, 3, 4.

(a) Write down the probability distribution table for X:

Possible value k	1	2	3	4
$P(X = k)$	$1/16$	$3/16$	$5/16$	$7/16$

(Hint: Drawing a four-by-four grid to show all possible outcomes of two rolls of a die might help)

~~[illegible]~~ First Roll

second roll	1	2	3	4
1	1	2	3	4
2	2	2	3	4
3	3	3	3	4
4	4	4	4	4

(b) What is the expected value $E(X)$ of X?

$$E[X] = 1 \times \frac{1}{16} + 2 \times \frac{3}{16} + 3 \times \frac{5}{16} + 4 \times \frac{7}{16}$$

$$= \frac{1}{16} + \frac{6}{16} + \frac{15}{16} + \frac{28}{16} = 50/16 = 25/8 = 3.125$$

(c) How likely is it, when you do this experiment, that you get an answer which is larger than the expected value?

We want $P(X \geq 3.125)$

$$= P(X = 4) = 7/16 = 0.4375$$

13 pts

12. ~~(12 pts.)~~ Last semester Prof. G. had 30 students in his class MATH 10860. On the first quiz, marked out of 5, here were the scores:

Score	0	1	2	3	4	5
Number of students with that score	0	3	7	9	9	2

(a) Draw a histogram representing this data.

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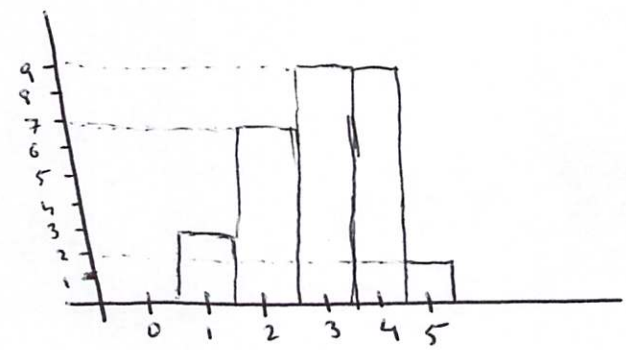

(b) Compute the mean score.

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$$\mu = (0\times 0 + 1\times 3 + 2\times 7 + 3\times 9 + 4\times 9 + 5\times 2)/30$$
$$= (0 + 3 + 14 + 27 + 36 + 10)/30$$
$$= 90/30 = 3$$

(c) Compute the standard deviation of the scores (*population* standard deviation).

3

x	freq	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 \times$ freq
0	0	-3	9	0
1	3	-2	4	12
2	7	-1	1	7
3	9	0	0	0
4	9	1	1	9
5	2	2	4	8

σ^2 = variance = $\dfrac{12+7+9+8}{30}$

$= 36/30 = 1.2$

σ = std dev = $\sqrt{1.2} \approx 1.095$

(d) What was the median score?

3

30 students, so median is the average of the 15^{th} and 16^{th} score (sorted by increasing order)

15^{th} score = 3
16^{th} score = 3 $\rightarrow$ median = 3.

13. (12 pts.) Alice plays the following game at the carnival. In each of eighteen rounds, she tosses a three sided die twice. She wins the round of the sum of the numbers that shows up is 3 or 5.

(a) What is the probability that Alice wins the first round of the game?

	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

$P(3 \text{ or } 5) = 4/9$

(b) What is the probability that Alice wins *exactly* sixteen rounds? For this part, you do not need to simplify your answer.

$X = \#$ rounds won

$\to X$ Binomial with $n = 18$, $p = 4/9$

$$P(X=16) = \binom{18}{16}(4/9)^{16}(5/9)^2$$

(c) What is the probability that Alice wins *at most* sixteen rounds? For this part, you do not need to simplify your answer.

$$P(X \leq 16) = 1 - P(X > 16)$$
$$= 1 - (P(X=17) + P(X=18))$$
$$= 1 - \left(\binom{18}{17}(4/9)^{17}(5/9) + (4/9)^{18}\right)$$

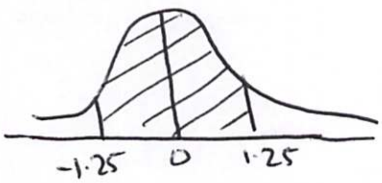
(d) If X is the number of rounds that Alice wins, what are the expected value and variance of X?

$$E[X] = np = 18 \times 4/9 = 8$$
$$Var(X) = npq = 18 \times 4/9 \times 5/9 = 40/9$$

14. (12 pts.) The length of newborn alligators, X, is normally distributed with mean $\mu = 7$ inches and standard deviation $\sigma = 1$ inches.

(a) What percentage of newborn alligators fall between 1.25 standard deviations below and 1.25 standard deviations above the mean length?

$$P(-1.25 \leq Z \leq 1.25) = A(1.25) + A(1.25)$$
$$= 0.3944 + 0.3944 = 0.7888$$

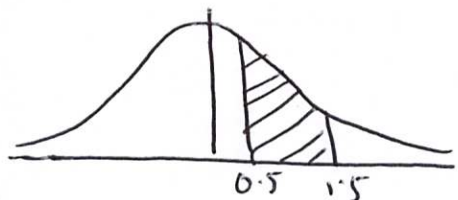


No need to convert because we are already given z-scores.

(b) Find the probability that a newborn alligator is between 7.5 and 8.5 inches long.

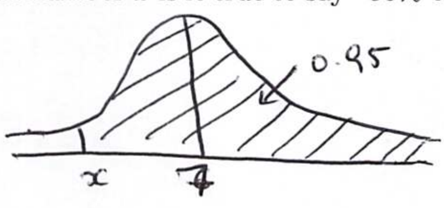
z-score of $7.5 = \frac{7.5-7}{1} = 0.5$

z-score of $8.5 = \frac{8.5-7}{1} = 1.5$



Probability $= A(1.5) - A(0.5)$
$= 0.4332 - 0.1915$
$= 0.2417$

(c) For which number x is it true to say "95% of the newborn alligators are at least x inches long"?



z-score of $x = \frac{x-7}{1} = x-7$

$A(z) + 0.5 = 0.95$

$A(z) = 0.45$

$\to z = -0.4495$ (or $z = -0.4505$)

$\to x = \cancel{0.6505}$ 6.5505 (or $x = 6.5495$)