The Honor Code is in effect for this examination. All work is to be your own.
• No calculators.
• The exam lasts for 1 hour and 15 min.
• Be sure that your name is on every page in case pages become detached.
• Be sure that you have all 9 pages of the test.

Please mark your answers with an X, not a circle!

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>2</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>3</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>4</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>5</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>6</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>7</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>8</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
</tbody>
</table>

Please do NOT write in this box.

Multiple Choice

9. 
10. 
11. 
12. 
Total
Multiple Choice

1. (7 pts.) Calculate \( \lim_{n \to \infty} \frac{(\ln n)^2}{n} \).

(a) 1  (b) \( \infty \)  (c) does not exist
(d) \( e^2 \)  (e) 0

2. (7 pts.) Find \( \sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}} \).

(a) \( \frac{5}{12} \)  (b) \( \frac{20}{3} \)  (c) \( \frac{5}{3} \)  (d) \( \frac{4}{15} \)  (e) \( \frac{5}{4} \)
3. (7 pts.) The series
\[ \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \]
(a) diverges because \( \lim_{n \to \infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0. \)
(b) diverges because the terms alternate.
(c) does not converge absolutely but does converge conditionally.
(d) diverges even though \( \lim_{n \to \infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0. \)
(e) converges absolutely.

4. (7 pts.) Use Comparison Tests to determine which one of the following series is divergent.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{n^3 + 1} \]
(b) \[ \sum_{n=1}^{\infty} \frac{n}{n + 1} \left( \frac{1}{2} \right)^n \]
(c) \[ \sum_{n=1}^{\infty} 7 \left( \frac{5}{6} \right)^n \]
(d) \[ \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^3 + 100} \]
(e) \[ \sum_{n=1}^{\infty} \frac{1}{n^2 + 8} \]
5. (7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\frac{x^2}{1 + x}$?

(a) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n + 2}$

(b) $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(d) $\sum_{n=0}^{\infty} x^{2n+2}$

(e) $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

6. (7 pts.) Find the degree 3 MacLaurin polynomial (Taylor polynomial centered at 0) for the function $\frac{e^x}{1 - x^2}$

(a) $1 + x + \frac{3x^2}{2} + \frac{7x^3}{6}$

(b) $1 + x - \frac{5x^3}{3}$

(c) $1 - \frac{x^2}{2} + \frac{x^3}{5}$

(d) $1 + x + \frac{x^2}{6} + 0 x^3$

(e) $1 + x - \frac{x^3}{6}$
7. (7 pts.) Which series below is a power series for \( \cos(\sqrt{x}) \) ?

(a) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!} \]

(c) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{n}}{n^2 + 1} \]

(d) \[ \sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{n}}{(2n)!} \]

(e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{n}}{(2n)!} \]

8. (7 pts.) Calculate

\[ \lim_{x \to 0} \frac{\sin(x^3) - x^3}{x^9} . \]

**Hint:** Without MacLaurin series this may be a long problem.

(a) \( \frac{9}{7} \)  (b) 0  (c) \( -\frac{1}{6} \)  (d) \( \infty \)  (e) \( \frac{7}{9} \)
9. (11 pts.) Does the series
\[ \sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}} \]
converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

Remark: The correct answer with no justification is worth 2 points.
10. (11 pts.) Use the Integral Test to discuss whether the series \( \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n} \) converges.

**Remark:** Be sure to check that the Integral Test can be applied. The correct answer with no justification is worth 2 points.
11. (11 pts.) Find the radius of convergence and interval of convergence of the power series
\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x - 3)^n \]

**Remark:** The correct answer with no justification is worth 2 points.
12. (11 pts.)
(a) Show that
\[ \sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1 + x^2} \]
provided that \( |x| < 1 \).

(b) Find
\[ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)(\sqrt{3})^{2n+1}}. \]
\textbf{(Hint:} First use term-by-term integration on the series in part (a).\textbf{)}
Math 10560, Practice Exam 3  
April 17, 2007

• The Honor Code is in effect for this examination. All work is to be your own.
• No calculators.
• The exam lasts for 1 hour and 15 min.
• Be sure that your name is on every page in case pages become detached.
• Be sure that you have all 9 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a)  (b)  (c)  (d)  (●)
2. (a)  (●)  (c)  (d)  (e)
3. (a)  (b)  (●)  (d)  (e)
4. (a)  (b)  (c)  (●)  (e)
5. (a)  (●)  (c)  (d)  (e)
6. (●)  (b)  (c)  (d)  (e)
7. (a)  (b)  (c)  (d)  (●)
8. (a)  (b)  (●)  (d)  (e)

Please do NOT write in this box.

Multiple Choice  

9.  
10.  
11.  
12.  

Total  