Name: ____________________________

Math 10560, Practice Final:
April 28, 2008

Instructor: ________________________

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2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
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9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(6 pts.) The function $f(x) = 2x + \ln x$ is one-to-one. Compute $(f^{-1})'(2)$.

(a) $\frac{1}{3}$ (b) $\frac{5}{2}$ (c) $4 + \ln 2$

(d) $\frac{2}{5}$ (e) 0

2.(6 pts.) Solve the equation $\log_4(x) + \log_4(x^2) = -\frac{3}{2}$. Then $x =$

(a) $\frac{1}{\sqrt{e}}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) (e) -2

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3.(6 pts.) Use logarithmic differentiation to compute the derivative of the function

$$f(x) = \frac{2^x(x^3+1)}{\sqrt{x+1}}.$$

(a) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\ln 2 + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

(b) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{2} + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

(c) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{\ln 2} + \dfrac{1}{x^3+1} - \dfrac{1}{x+1}\right)$

(d) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(2 + \dfrac{1}{x^3+1} - \dfrac{1}{x+1}\right)$

(e) $f'(x) = \dfrac{2^x(x^3+1)}{\sqrt{x+1}}\left(\dfrac{1}{\ln 2} + \dfrac{3x^2}{x^3+1} - \dfrac{1}{2(x+1)}\right)$

4.(6 pts.) You begin an experiment at 9am with a sample of 1000 bacteria. An hour later your population has doubled. Assuming exponential growth, what is the population at noon?

(a) $32,000$ (b) $4,000$ (c) $8,000$ (d) $1,000e^{-3}$ (e) $1,000e^{3}$

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5.(6 pts.) Simplify $\sin^{-1}(\sin \frac{9\pi}{10})$.

(a) $-\frac{\pi}{10}$

(b) $\frac{9\pi}{10}$

(c) not enough information to tell.

(d) $\frac{\pi}{10}$

(e) 0

6.(6 pts.) Compute the limit $\lim_{x\to\infty} (2x)^{\frac{1}{x}}$.

(a) 1 (b) 0 (c) e (d) 2 (e) ∞

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7.(6 pts.) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} x\sin(x)dx.$$

(a) 0 (b) 1 (c) -1 (d) π (e) $\dfrac{\pi}{2}$

8.(6 pts.) Find the integral $\int_0^2 \sqrt{4-x^2}dx$.

(a) 0 (b) $4+\sin 4$ (c) π (d) $\pi+2$ (e) -2

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9.(6 pts.) Evaluate the integral

$$\int \frac{x-9}{x^2+3x-10}\,dx.$$

(a) $\ln\left|\frac{(x-5)^2}{x+2}\right| + C$ (b) $\ln\left|\frac{x-2}{x+5}\right| + C$ (c) $\ln\left|\frac{(x+2)^2}{(x-5)^3}\right| + C$

(d) $\ln\left|\frac{(x+5)^2}{x-2}\right| + C$ (e) $\ln\left|\frac{x+5}{(x-2)^2}\right| + C$

10.(6 pts.) Determine whether the following integral converges or diverges. If it converges, evaluate.

$$\int_{-2}^{0} \frac{1}{(x+1)^2}\,dx.$$

(a) Converges to -2. (b) Converges to 0. (c) Converges to 2.

(d) Diverges. (e) Converges to 1.

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11.(6 pts.) Find the length of the curve

$$y = 1 + \frac{4}{3}x^{3/2} \qquad \text{for } 0 \le x \le 1.$$

(a) $\frac{1}{6}(1 - \sqrt{5})$ (b) $\frac{2}{3}(1 - \sqrt{5})$ (c) $\frac{1}{12}(3\sqrt{3} - 1)$

(d) $\frac{1}{12}(11\sqrt{5} - 1)$ (e) $\frac{1}{6}(5\sqrt{5} - 1)$

12.(6 pts.) Find the centroid of the region bounded by $y = x^2$ and $y = x$.

(a) $(\frac{1}{2}, \frac{1}{2})$ (b) $(\frac{1}{10}, \frac{2}{5})$ (c) $(\frac{1}{10}, \frac{1}{15})$

(d) $(\frac{1}{12}, \frac{1}{15})$ (e) $(\frac{1}{2}, \frac{2}{5})$

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13.(6 pts.) The solution to the initial value problem

$$y' = \frac{\sin(x)}{2y+1} \qquad\qquad y(0) = 2$$

satisfies

(a) $2y + 1 = 6 - e^{-\cos x}$

(b) $y^2 + y = 7 - \cos x$

(c) $y^2 + y = 6\cos x$

(d) $2y + 1 = 5e^{-\cos x}$

(e) $e^{2y+1} = e^5 + \arcsin x$

14.(6 pts.) The solution to the initial value problem

$$\frac{dy}{dx} + xy + x = 0 \qquad y(0) = 0$$

is

(a) $y = e^{-\frac{x^2}{2}} - 1$

(b) $y = e - e^{-\frac{x^2}{2}+1}$

(c) $y = xe^x$

(d) $y = 1 - e^{-x}$

(e) $y = e^{-x} - 1$

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15.(6 pts.) Investigate the convergence or divergence of the sequence

$$\lim_{n\to\infty} (-1)^n \frac{3n^2}{n^2+1}.$$

If the sequence converges, find its limit.

(a) -3 (b) $(-1)^n 3$ (c) The sequence is divergent

(d) 3 (e) 0

16.(6 pts.) Investigate convergence or divergence of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(4-\pi)^{n-1}}{\pi^n}.$$

If the series converges, calculate its sum. Note: $4 > \pi > 3$.

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) The series is divergent

(d) $\frac{1}{4}$ (e) $-\frac{1}{4}$

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17.(6 pts.) The series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cos\left(\frac{1}{n}\right)$$

is

(a) absolutely convergent by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(b) conditionally convergent by root test

(c) divergent by integral test

(d) divergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(e) absolutely convergent by ratio test

18.(6 pts.) Which of the following series converge conditionally?

$$(1) \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n}+1}; \qquad (2) \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}+1}; \qquad (3) \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^{5/2}+1}.$$

(a) (1) and (2) converge conditionally, (3) does not converge conditionally

(b) (2) converges conditionally, (1) and (3) do not converge conditionally

(c) (1) converges conditionally, (2) and (3) do not converge conditionally

(d) (1) and (3) converge conditionally, (2) does not converge conditionally

(e) (3) converges conditionally, (1) and (2) do not converge conditionally

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19.(6 pts.) The series

$$\sum_{n=1}^{\infty} \frac{8^n}{n^2}(x-1)^{3n}$$

has the radius of convergence

(a) 0 (b) $\frac{1}{2}$ (c) ∞ (d) 1 (e) $\frac{1}{8}$

20.(6 pts.) Consider the Taylor series of

$$f(x) = \sum_{n=1}^{\infty} \frac{n^n}{n!} x^n.$$

Find $f^{(100)}(0)$.

(a) $\frac{100^{100}}{((100)!)^2}$ (b) $\frac{(100)!}{100^{100}}$ (c) $\frac{100^{100}}{(100)!}$

(d) $(100)!$ (e) 100^{100}

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21.(6 pts.) Which is the only statement that is true about the three series

$$\text{(I)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+n^2} \quad \text{(II)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{1+n^2} \quad \text{(III)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln n}{1+n^2} \ ?$$

(a) (I) and (III) converge conditionally, (II) converges absolutely

(b) (I) diverges, (II) converges conditionally, (III) converge absolutely

(c) (I) and (II) converge absolutely, (III) converges conditionally

(d) (I) and (III) converge conditionally, (II) diverges

(e) (I) and (III) converge absolutely, (II) converges conditionally

22.(6 pts.) Let $x = \sin(9t)$ and $y = \cos(9t)$. Then $\dfrac{dy}{dx} =$

(a) $\tan(9t)$ (b) $-\tan(9t)$ (c) $9\tan(t)$

(d) $81\sec^2(9t)$ (e) $\cot(9t)$

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23.(6 pts.) The point $(2, \dfrac{13\pi}{6})$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(\sqrt{3}, 1)$

(b) $(-\sqrt{3}, 1)$

(c) $(1, \sqrt{3})$

(d) $(-1, \sqrt{3})$

(e) Since $\dfrac{13\pi}{6} > 2\pi$, there is no such point.

24.(6 pts.) Which integral below gives the surface area of the surface of revolution obtained by rotating the polar curve $r = \sin\theta$, $0 \le \theta \le \pi$ about the x-axis?
Hint: A polar curve is also a parameterized curve.

(a) $2\pi \displaystyle\int_0^{\pi} \sin\theta \ \cos^2\theta \, d\theta$

(b) $2\pi \displaystyle\int_0^{\pi} \cos^2\theta \, d\theta$

(c) $2\pi \displaystyle\int_0^{\pi} \sin^2\theta \, d\theta$

(d) $\dfrac{\pi}{2} \displaystyle\int_0^{\pi} \sin\theta \ \cos^2\theta \, d\theta$

(e) $2\pi \displaystyle\int_0^{\pi} \sin\theta \ \cos\theta \, d\theta$

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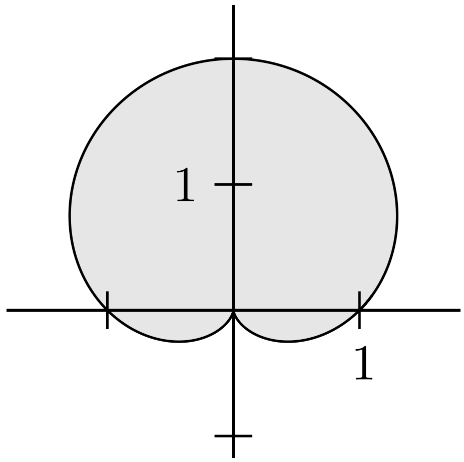
25.(6 pts.) Find the area inside the cardioid $r = 1 + \sin\theta$.



(a) $\dfrac{3}{2}$ (b) 2π (c) 2 (d) $3\pi + \ln 4$ (e) $\dfrac{3\pi}{2}$

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