Math 10850, Honors Calculus 1

Quiz 2, Thursday September 12

Name:

1. Show that if a, x and y are real numbers satisfying $x + a = y + a$, then $x = y$. Use *only* the axioms of the real numbers, and the standard properties of equality; don't re-use anything that we proved in class. Say which axiom/axioms you are using at each step (the relevant ones appear overleaf to help you out).

2. Show that if a is any real number then $a \cdot 0 = 0$. Same rules apply as for part 1, *except* you can also use the result of part 1, if you wish.

3. A consequence of part 1 is that the additive inverse of P3 is unique. Using *only* the axioms of the real numbers, the standard properties of equality, and (if necessary) the uniqueness of inverses & the result of part 2, prove that for all real x, $-x = (-1)x$. (**NB**: this has nothing to do with positivity, order, and/or P10 through P12.)

Axioms of the real numbers

The real numbers, denoted $\mathbb{R}$, is a set (of numbers) with

- two special numbers, 0 and 1, that are distinct from each other,
- an operation $+$, *addition*, that combines numbers a, b to form the number $a + b$,
- an operation $\cdot$, *multiplication*, that combines a, b to form $a \cdot b$, and
- a set $\mathbb{P}$ of *positive* numbers,

that satisfies the following 13 axioms:

P1, Additive associativity For all a, b, c, $\boxed{a + (b + c) = (a + b) + c}$.

P2, Additive identity For all a, $\boxed{a + 0 = 0 + a = a}$.

P3, Additive inverse For all a there's a number $-a$ with $\boxed{a + (-a) = (-a) + a = 0}$.

P4, Additive commutativity For all a, b, $\boxed{a + b = b + a}$.

P5, Multiplicative associativity For all a, b, c, $\boxed{a \cdot (b \cdot c) = (a \cdot b) \cdot c}$.

P6, Multiplicative identity For all a, $\boxed{a \cdot 1 = 1 \cdot a = a}$.

P7, Multiplicative inverse For all a, if $a \neq 0$ there's a number a^{-1} such that $\boxed{a \cdot a^{-1} = a^{-1} \cdot a = 1}$.

P8, Multiplicative commutativity For all a, b, $\boxed{a \cdot b = b \cdot a}$.

P9, Distributivity of multiplication over addition For all a, b, c, $\boxed{a \cdot (b + c) = (a \cdot b) + (a \cdot c)}$.

P10, Trichotomy law

P11, Closure under addition

P12, Closure under multiplication

P13, Completeness