Math 10850, Honors Calculus 1

Quiz 7, Thursday October 31

Solutions

1. Give a complete and correct statement of the *Intermediate value theorem* (IVT)

   Solution: If $f : [a, b] \to \mathbb{R}$ is continuous (implicit in this: continuous at every point in the open interval (a, b), right continuous at a and left continuous at b, though all of this doesn't need to be said), and if $f(a) < 0$ and $f(b) > 0$, then there is $c \in (a, b)$ with $f(c) = 0$.

   An alternative: if f is a continuous function defined on a closed interval, and f is negative at one end of the interval and positive at the other end of the interval, then there is some point in the interval where f is zero.

   Another alternative: if f is a continuous function defined on an interval, and f takes on two different values, say x_1 and x_2, then f takes on every value between x_1 and x_2.

2. Use the intermediate value theorem to show that there is some real number c such that $c^5 = c + 1$. (You should use the theorem directly, and not use any consequences that we have derived from the theorem, such as facts about odd-degree polynomials).

   Solution: One way to approach this question is to define the function $f(x) = x^5 - x - 1$. This is a continuous function on the closed interval $[0, 2]$. We have $f(0) = -1 < 0$ and $f(2) = 32 - 2 - 1 = 29 > 0$. By the intermediate value theorem, there is $c \in (0, 2)$ with $f(c) = 0$, i.e., with $c^5 - c - 1 = 0$, or $c^5 = c + 1$, as required.