1. Let $f : [a, b] \to \mathbb{R}$, with $a < b$.

   (a) What hypothesis do we need to put on $f$, in order to be able to define upper and lower Darboux sums for all partitions of $[a, b]$?

(b) Let $P = t_0, t_1, \ldots, t_n$ be a partition of $[a, b]$, with $a = t_0 < t_1 < t_2 < \cdots < t_n = b$. What is the lower Darboux sum $L(f, P)$? Define any new symbols you might use in your answer.

2. Suppose that $f$ is integrable on $[a, b]$, for some $a < b$, and suppose that $c$ and $d$ are such that $a < c < d < b$. Show that $f$ is integrable on $[c, d]$. (In this question you may use, without proof, any of the basic properties of integration that we established in class on Wednesday, as long as you state the properties clearly and correctly. Spivak’s hint for this question is “don’t work too hard”).