Math 10860, Honors Calculus 2

Midterm 2 information

Spring 2020

The second midterm will be

Friday April 17.

It will be a 60 minute exam, but I will give you 90 minutes to do it. Here's how it will work:

- At some point between 11.30am and 12.30pm SBT, check in with me at `https://notredame.zoom.us/j/844811786`.
- Once you have checked in, I will share the exam with you at your ND email address (as a pdf file in a Google drive).
- From the time I share the exam with you, you have 90 minutes to do the it (on your own paper). When you are done, scan the exam to a single pdf file, and email it back to me.
- Unless you let me know that you are having technical problems, there will be a penalty for returning the exam late.
- I'll be on Zoom from 11.30am through 1.00pm SBT, to answer questions and give clarifications. Also, from 1.00pm to 2.00pm, I'll easily be contactable by email.
- Remember: start the exam sometime between 11.30am and 12.30pm SBT, by checking in with me; finish within 90 minutes.

The exam will cover everything from class on Wednesday, February 26 through Wednesday April 8: methods of integration (parts, substitution, trigonometric substitution, trigonometric integrals, partial fractions); Taylor things (agreement, Taylor polynomials, remainder terms, Taylor's theorem); sequences (basic properties, relation to continuity, bounded monotone sequences). In the course notes, that is Chapters 13, 14 and 15 (for Chapter 15, only through to the top of page 373 — so not subsequences and Bolzano-Weierstrass); in terms of homeworks and quizzes, it is everything that was covered on homeworks 7 through 9 and quizzes 7 through 9.

Because it is essentially a take-home exam, the focus will be more on problems than definitions/statements of theorems.

Practice problems

1. Let
$$I_n = \int x^n \sin x \, dx.$$

   (a) Find I_0 and I_1.

   (b) Find a reduction formula that expresses I_{n+2} in terms of I_n for $n \geq 0$.

   (c) Find $\int x^5 \sin x \, dx$.

2. Find the following integrals:

   (a)
$$\int \log^3 x \, dx.$$

   (b)
$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}} \, dx.$$

   (c)
$$\int \frac{dx}{2+\tan x}.$$

   (d)
$$\int \frac{x^6 + x^5 - 2x^4 - x^3 + 8x^2 - 4x + 5}{(x-1)^2(x+1)^3} \, dx.$$

   (e)
$$\int_0^{\pi} (f(x) + f''(x)) \sin x \, dx.$$

   (Here f is defined and twice differentiable on $[0, \pi]$, with f'' continuous. Your answer will depend on f, of course.)

3. Recall that $\sinh x = \frac{e^x - e^{-x}}{2}$.

   (a) Find the degree $2n+1$ Taylor polynomial of sinh about 0.

   (b) Write down the Lagrange form of the remainder term $R_{2n+1,0,\sinh}(x)$.

   (c) Show that for all real x the remainder term $R_{2n+1,0,\sinh}(x)$ tends to 0 as n tends to infinity.

   (d) Write down a sum (using summation notation), with all the summands being rational numbers, whose value is within 10^{-10} of $\sinh 5$.

4. Suppose that a_i is the coefficient of $(x-a)^i$ in the Taylor polynomial of $f(x)$ at a (so $a_i = \frac{f^{(i)}(a)}{i!}$) and that b_i is the coefficient of $(x-a)^i$ in the Taylor polynomial of $g(x)$ at a. In terms of the a_i's and the b_i's, express the coefficient of $(x-a)^n$ in the Taylor polynomial of each of the following functions at a:

(a) $2f - 3g$

(b) fg.

(c) $h(x) = \int_a^x f(t)\ dt$.

5. Compute the following sequence limits:

(a)

$$\lim_{n\to\infty} \frac{a^n - b^n}{a^n + b^n}.$$

(Here a, b are arbitrary real constants; you may have to treat cases.)

(b)

$$\lim_{n\to\infty} (n - \sqrt{n-a}\sqrt{n-b}).$$

(Again, a, b are arbitrary real constants; you may have to treat cases.)

6. Define two sequences $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ recursively by $0 < a_1 < b_1$ (some arbitrary reals) and for $n \geq 1$

$$a_{n+1} = \sqrt{a_n b_n}, \quad b_{n+1} = \frac{a_n + b_n}{2}.$$

(a) Prove that $b_n \geq a_n$ for all $n \geq 1$.

(b) Prove that $(a_n)_{n=1}^{\infty}$ is non-decreasing.

(c) Prove that $(b_n)_{n=1}^{\infty}$ is non-increasing.

(d) Explain why $(a_n)_{n=1}^{\infty}$, $(b_n)_{n=1}^{\infty}$ both converge to finite limits.

(e) Show that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$.