Math 10860, Honors Calculus 2

Quiz 1, Thursday January 23

Solutions

1. Let $f : [a, b] \to \mathbb{R}$, with $a < b$.

   (a) What (single) additional hypothesis do we need to put on f, in order to be able to define upper and lower Darboux sums for all partitions of $[a, b]$?[1]

   Solution: f needs to be *bounded* for the definition of upper and lower Darboux sums to make sense for every partition of $[a, b]$

   (b) Let $P = t_0, t_1, \ldots, t_n$ be a partition of $[a, b]$, with $a = t_0 < t_1 < t_2 < \cdots < t_n = b$. What is the *lower Darboux sum* $L(f, P)$? Define any new symbols you use in your answer.

   Solution: $L(f, P) = \sum_{i=1}^{n} m_i \Delta_i$, where $m_i = \inf\{f(x) : x \in [t_{i-1}, t_i]\}$ and $\Delta_i = t_i - t_{i-1}$.

2. (a) Suppose that f is integrable on $[a, b]$, for some $a < b$, and suppose that c and d are such that $a < c < d < b$. Show that f is integrable on $[c, d]$. (In this question you may use, without proof, any of the basic properties of integration that we established in class, as long as you state the properties clearly and correctly. If space is tight, you can use the back to give the property statements.)

   Solution: f is integrable on $[a, b]$, and $a < c < b$, so by one of the basic properties that we saw in class on Friday — f integrable on $[a, b]$ and $a < c < b$ implies f integrable on both $[a, c]$ and $[c, b]$ — we know that f is integrable on $[c, b]$. Now $c < d < b$, so by the same property we know that f is integrable on $[c, d]$.

   (b) The full-blown definition of the expression $\int_a^b f$ was given for $a < b$. What is the correct way to interpret $\int_a^b f$ when $a > b$?

   Solution: For $a > b$, we interpret $\int_a^b f$ to be $-\int_b^a f$, **IF** $\int_b^a f$ exists. If $\int_b^a f$ does not exist, then we attach no meaning to $\int_a^b f$.

[1] Note that by writing "$f : [a, b] \to \mathbb{R}$", I'm already hypothesizing that f is defined at all points between a and b.