Statistics for the Life Sciences

Math 20340 Section 01, Fall 2008

Homework 8 Solutions

- **7.6**: This one is open to interpretation; there are surely a variety of "correct" answers. My feeling is that people who are strongly opposed to a plan are generally more vocal than those who have no objection to it; so probably among the 500 non-respondents, a majority of them are not opposed to the surcharge, meaning that .72 is probably an overestimate of the proportion of the population who oppose the surcharge.
- **7.11**:
  - **a**: This is what is referred to in the text as a "convenience sample": a sample chosen because it presents an easy opportunity for the researchers to gather data.
  - **b**: No chance mechanism was used — this is *not* a random sample.
  - **c**: Possibly not ... is there any good reason to believe that the Native students at one particular school in MN should be representative of urban dwelling Native American teens across the whole country?
  - **d**: A better plan would at least include teens from a variety of different urban areas.
- **9.3**:
  - **a**: $z > 2.33$
  - **b**: $z > 1.96$ or $z < -1.96$ (also could be written as $|z| > 1.96$)
  - **c**: $z < -2.33$
  - **d**: $z > 2.58$ or $z < -2.58$ (also could be written as $|z| > 2.58$)
- **9.6**:
  - **a**: $H_0 : \mu = 2.3$ (or, equally appropriate, $H_0 : \mu \leq 2.3$) versus $H_a : \mu > 2.3$
  - **b**: Reject null if test statistic $\frac{\bar{x}-2.3}{.29/\sqrt{35}} > 1.64$. This is same as: reject null if $\bar{x} > 2.38...$
  - **c**: $s = .29/\sqrt{35} \approx .049$
  - **d**: My intuition was that 2.4 is a reasonable value. But test statistic is 2.04..., so in fact we should reject null at 5% level of significance
- **9.7**:

- **a**: p-value is $P(z > 2.04) = .0207$
- **b**: Reject H_0 (p-value is less than .05)
- **c**: Yes

- **9.8**:

  - **a**: At 5% significance, as we've previously observed, null would be rejected as long as $\bar{x} > 2.38...$
  - **b**: We would accept H_0 if $\bar{x} \leq 2.38$. So we want to compute $P(\bar{x} \leq 2.38)$, given that $\mu = 2.4$, that is, given that $\bar{x}$ is a normal random variable with mean 2.4, SE .049. Normalizing, this is $P(z \leq (2.38 - 2.4)/.049 = -.4) = .3446$.
  - **c**: For $\beta = 2.3$, we are looking at $P(z \leq (2.38 - 2.3)/.049) = P(z \leq 1.63) = .9485$. (This answer should have been exactly .95, but I've mae some small rounding error using 2.38 instead of 2.38...). For $\beta = 2.5$, we are looking at $P(z \leq (2.38 - 2.5)/.049) = P(z \leq -2.45) = .0071$. For $\beta = 2.6$, we are looking at $P(z \leq (2.38 - 2.6)/.049) = P(z \leq -.449) = 0$.
  - **d**: The power $1 - \beta$ increases from close to 5% when μ is close to 2.3, to close to 100% when $mu = 2.6$.

- **9.12**:

  - **a**: We want to test $H_0 : \mu = 5$ versus $H_a : \mu \neq 5$. Test statistic is $(11.17 - 5)/(3.9/\sqrt{50}) = 11.18$. Since this is (much) greater than 1.96, we reject H_0 at 5% significance level.
  - **b**: p-value is $P(z > 11.18) = 0$, so using this we also reject at 5% significance level.

- **9.13**:

  - **a**: $H_0 : \mu = 80$
  - **b**: $H_a : \mu \neq 80$
  - **c**: Test statistic $(79.7 - 80)/(.8/\sqrt{100}) = -3.75$. Since this is less that -1.96, we *reject* the null at 5% significance; there is evidence to suggest that the potency is not 80%

- **9.14**: $H_0 : \mu = 7$; $H_a : \mu < 7$. Test statistic is $(6.7 - 7)/(2.7/\sqrt{80}) = -.99...$. This is *not* below -1.645, so there is *not* evidence to reject the null at 5% significance.

- **9.17**: $H_0 : \mu = 5.97$; $H_a : \mu > 5.97$. Test statistic is $(9.8 - 5.97)/(1.95/\sqrt{31}) = 10.93$. This is (much) greater that 1.645, so there is strong evidence to reject null at 5% significance.

- **9.18**:

  - **a**: $H_0 : \mu_1 = \mu_2$; $H_a : \mu_1 > \mu_2$
  - **b**: One-tailed

- **c**: Test statistic is $\frac{\bar{x}_1}{-}\bar{x}_2\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 2.087...$
- **d**: p-value is $P(z > 2.08) = .0188$. At 1% significance, there is *not* evidence to suggest a difference
- **e**: Rejection region is $TS > 2.33$. Since test statistic is 2.08, there is not evidence to reject null

- **9.20**:

  The described process (deciding on which test to use based on the observed date) doubles the probability of type I error to .1.

  If H_0 is true, that is, if $\mu_1 = \mu_2$, then half the time we will have $\bar{x}_1 > \bar{x}_2$ and half the time we will have $\bar{x}_2 > \bar{x}_1$ (I'm ignoring the possibility that $\bar{x}_1 = \bar{x}_2$, which will happen very infrequently). Let's focus on what happens if we find that $\bar{x}_1 > \bar{x}_2$. In this case, the proposed plan is to run the one sided test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 > \mu_2$ at significance level α. We want to figure out what is the probability that we reject H_0 given that it is true. Since this is a one-sided test, this is $P(TS > z_\alpha)$, which we might think is just α (that, after all, is the definition of the significance α). *BUT*, we are computing this probability conditioned on some extra information, namely, the information that $\bar{x}_1 > \bar{x}_2$. So actually the probability of rejecting H_0 in this case is $P(TS > z_\alpha | \bar{x}_1 > \bar{x}_2) = P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2)/P(\bar{x}_1 > \bar{x}_2)$. Since $P(\bar{x}_1 > \bar{x}_2) = .5$, this is $2P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2)$. If we know that $TS > z_\alpha$, then we automatically know that $\bar{x}_1 > \bar{x}_2$; so $P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2) = P(TS > z_\alpha) = \alpha$, and the probability of rejecting H_0 when it is true in this case (the case $\bar{x}_1 > \bar{x}_2$) is 2α, not α. In the other case (the case $\bar{x}_1 < \bar{x}_2$), the probability of rejecting H_0 when it is true is also 2α. Either way, the probability of type I error is 2α, which is .1 if $\alpha = .05$.

- **9.23**:

  - **a**: The test statistic is -2.26. The p-value is $.0238$ (remember that this is two-tailed test); we should reject H_0 at 5% significance
  - **b**: $(-3.55, -.25)$ is the 95% confidence interval (note that 0 is not in this interval, which is why we rejected H_0)
  - **c**: Since -5 (and 5) is not in the confidence interval, the difference between the two means is not of practical importance

- **9.27**:

  - **a**: The test statistic is -3.18. The p-value is $.0014$ (remember that this is two-tailed test); there is sufficient evidence to reject H_0 at 5% significance and conclude that there is a difference
  - **b**: $(-3.01, -.71)$ is the 95% confidence interval. This agrees with the previous part; 0 is not in this interval