Air Force transportation problem

The Air Force wants to introduce a new shielding material for its B-52 planes. An initial test phase will involve planes at five bases: March (CA), Davis (AZ), McConnell (KS), Pinecastle (FL) and McDill (FL). Supplies are available at three depots: Columbus (OH), Oklahoma City (OK), Macon (GA). The distance between depots and bases (in miles) is shown in the table below, together with the number of tons of the material needed at each base and available at each depot (shown in parentheses by each location). See Figure 5 for a map.

	McDill (3)	**March** (5)	**Davis** (5)	**McConnell** (5)	**Pinecastle** (3)
OK City (8)	938	1030	824	136	995
Macon (5)	346	1818	1416	806	296
Columbus (8)	905	1795	1590	716	854

An LP formulation

This is an example of a *transportation problem*: material has to be transported from a collection of sources to a collection of destinations. We set up fifteen variables, x_{ij}, $i = 1, \ldots, 3$, $j = 1, \ldots, 5$, with x_{ij} representing the amount of material shipped from source i to destination j. (Here we number the sources and destinations according to their appearance in the columns and rows of the table on the previous page).

What will be the objective function? In the absence of information on the various transportation costs, we could try to minimize the total miles traveled, but that does not take into account the clear fact that the more that is being transported, the greater the cost. To take this into account, we choose to minimize total *ton-miles*.

The LP problem

We seek to minimize

$$938x_{11} + 1030x_{12} + 824x_{13} + 136x_{14} + 995x_{15} + 346x_{21} + \ldots + 854x_{35}$$

subject to $x_{ij} \geq 0$ for all i, j, and the supply and demand constraints

$$\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &\leq 8 \\
x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &\leq 5 \\
x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &\leq 8 \\
x_{11} + x_{21} + x_{31} &\geq 3 \\
x_{12} + x_{22} + x_{32} &\geq 5 \\
x_{13} + x_{23} + x_{33} &\geq 5 \\
x_{14} + x_{24} + x_{34} &\geq 5 \\
x_{15} + x_{25} + x_{35} &\geq 3.
\end{aligned}$$

Solving the LP

A first guess: It seems sensible to make use of the proximity of McConnell to Oklahoma, and of Pinecastle and McDill to Macon. Sending as much as possible from each depots to the closest bases (referred to as a *greedy* approach) leads to the solution shown in Figure 6:

$$x_{13} = 3, x_{14} = 5, x_{21} = 2, x_{25} = 3, x_{31} = 1, x_{32} = 5, x_{33} = 2,$$

with all other $x_{ij} = 0$. This leads to a total of 17,792 ton-miles.

The optimal solution: The following is the actual optimal:

$$x_{12} = 3, x_{13} = 5, x_{21} = 3, x_{25} = 2, x_{32} = 2, x_{34} = 5, x_{35} = 1,$$

with all other $x_{ij} = 0$. This leads to a total of 16,864 ton-miles. (See Figure 7). Note that although this solution eliminates completely the short leg from OK City to McConnell, it more than halves the tonnage on the longest possible leg, Columbus to March.