A blending problem (Taha, Example 2.3-7, almost)

An oil refinery has three stages of production: a distillation tower, which takes in crude oil, up to a maximum of 650,000 barrels per day (bbl/day) and produces feedstock with octane rating 82 ON at a rate of .2 bbl per bbl of crude; a cracker unit which takes in feedstock (maximum 200,000 bbl/day) and produces gasoline stock with 98 ON at a rate of .5 bbl per bbl of feedstock; and a blender unit which blends feedstock and gasoline stock (at no loss). (Note that "ON **" means "**% octane".) Once crude oil enters the system, it goes fully through the process. The refinery aims to produce regular gas (87 ON at least, maximum daily demand 50000 bbl, profit $6.70/bbl), premium (89 ON, 30,000 bbl, $7.20/bbl) and super (92 ON, 40,000 bbl, $8.10/bbl).

Produce a refining schedule that maximizes profit.

Note: Taha's Example 2.3-7 has distillation tower capacity 1,500,000 bbl/day, and is otherwise the same.

Solution

Let x_1 be daily input to distillation tower

Constraint: $0 \leq x_1 \leq 650000$

Output from distillation tower is $x_1/5$

Let x_2 and x_3 be amounts of feedstock fed into blender and cracker unit

Constraints: $0 \leq x_2,\ 0 \leq x_3 \leq 200000,\ x_2 + x_3 = x_1/5$

Output from cracker unit to be fed into blender as gasoline stock is $x_3/2$

Let a_1, a_2, a_3 be amounts of feedstock used for regular, premium, super

Let b_1, b_2, b_3 be amounts of gas. stock used for regular, premium, super

Constraints: all $a_i, b_i \geq 0,\ a_1 + a_2 + a_3 = x_2,\ b_1 + b_2 + b_3 = x_3/2$

Regular ON constraint: $82a_1 + 98b_1 \geq 87(a_1 + b_1)$

Premium ON constraint: $82a_2 + 98b_2 \geq 89(a_2 + b_2)$

Super ON constraint: $82a_3 + 98b_3 \geq 92(a_3 + b_3)$

Demand constraints:

$a_1 + b_1 \leq 50000,\ a_2 + b_2 \leq 30000,\ a_3 + b_3 \leq 40000$

Objective: maximize $6.7(a_1 + b_1) + 7.2(a_2 + b_2) + 8.1(a_3 + b_3)$

TORA-friendly formulation

Maximize

$$6.7a_1 + 6.7b_1 + 7.2a_2 + 7.2b_2 + 8.1a_3 + 8.1b_3$$

subject to

$$\begin{aligned}
x_1 &\leq 650000 \\
x_3 &\leq 200000 \\
-.2x_1 + x_2 + x_3 &= 0 \\
-x_2 + a_1 + a_2 + a_3 &= 0 \\
-.5x_3 + b_1 + b_2 + b_3 &= 0 \\
-5a_1 + 11b_1 &\geq 0 \\
-7a_2 + 9b_2 &\geq 0 \\
-10a_3 + 6b_3 &\geq 0
\end{aligned}$$

$$\begin{aligned} a_1 + b_1 &\leq 50000 \\ a_2 + b_2 &\leq 30000 \\ a_3 + b_3 &\leq 40000 \\ \text{and all variables} &\geq 0. \end{aligned}$$

A solution via TORA:

$$\begin{array}{ccc} x_1 = 650,000 & x_2 = 56,154 & x_3 = 73,846 \\ a_1 = 34,375 & a_2 = 16,875 & a_3 = 4,904 \\ b_1 = 15,625 & b_2 = 13,125 & b_3 = 8,173 \end{array}$$

This solution proposes producing 50,000 barrels of regular and 30,000 barrels of premium (meeting the demand), but only 13,077 barrels of super, for a profit of $656,923.