Lawyer problem

A legal firm has accepted five new cases, each of which can be adequately handled by any one of the firms five junior partners. Due to differences in experience and expertise, the junior partners would spend varying amounts of time on the cases. A senior partner has estimated the time requirements (in hours), and this information is tabulated below. Set up as a linear program the problem of the senior partner assigning cases, one per junior lawyer, in such a way as to minimize the total hours expended.

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
Lawyer 1	145	122	130	95	115
Lawyer 2	80	63	85	48	78
Lawyer 3	121	107	93	69	95
Lawyer 4	118	83	116	80	105
Lawyer 5	97	75	120	80	111

Solution

We set up 25 variables, x_{ij}, $i = 1, \ldots 5$, $j = 1, \ldots, 5$, with x_{ij} the "amount" of case i assigned to lawyer j. Because lawyers and cases can't be split, each of the x_{ij} must be either 0 or 1.

We need to minimize lawyer-hours

$$145x_{11} + 80x_{12} + 121x_{13} + 118x_{14} + 97x_{15} + 122x_{21} + \ldots + 111x_{55}$$

subject to the constraint that Case 1 is actually handled:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1$$

and similar constraints for Cases 2 through 5, as well as the constraint that Lawyer 1 receives exactly one case:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1$$

and similar constraints for Lawyers 2 through 5. Finally, we have the implicit constraints that all $x_{ij} \geq 0$ and all are integers.