Meatloaf problem

A butcher has pork and beef available to him to make meatloafs. Each meatloaf should weight at least 1 kilo. The customers demand that the fat content of a meatloaf should be no more than 25%. The beef that the butcher works with contains 20% fat, and the pork contains 32% fat. If the beef costs 80 cents per kilo and the pork costs 60 cents, how should the butcher mix beef and pork in the meatloaf in order to minimize his cost?

A equivalent linear programming problem?

Let x be kilos of beef used in each meatloaf, and y be kilos of pork used. Since the butcher is trying to minimize his cost, it's reasonable (?) to assume that he should make meatloafs that are exactly 1 kilo, so that we should have $x + y = 1$.

Minimize

$$80x + 60y$$

subject to

$$\begin{aligned} x + y &= 1 \\ .2x + .32y &\leq .25 \\ x,\ y &\geq 0. \end{aligned}$$

A more correct formulation

Let x be kilos of beef used in each meatloaf, and y be kilos of pork used.

Minimize

$$80x + 60y$$

subject to

$$\begin{aligned} x + y &\geq 1 \\ .2x + .32y &\leq .25(x + y) \\ \text{(i.e., } -.05x + .07y &\leq 0) \\ x,\ y &\geq 0. \end{aligned}$$

Why? An additional constraint may eliminate all feasible solutions with $x + y = 1$. E.g., the butcher, knowing his daily demand and the expiry date of some of his pork, may determine that he must use a minimum .6 kilos of pork per meatloaf. Note that .4 kilo beef and .6 kilo pork already has 272 grams of fat, so need to go above 1 kilo to get below 25% fat.

Solution to first formulation (via TORA)

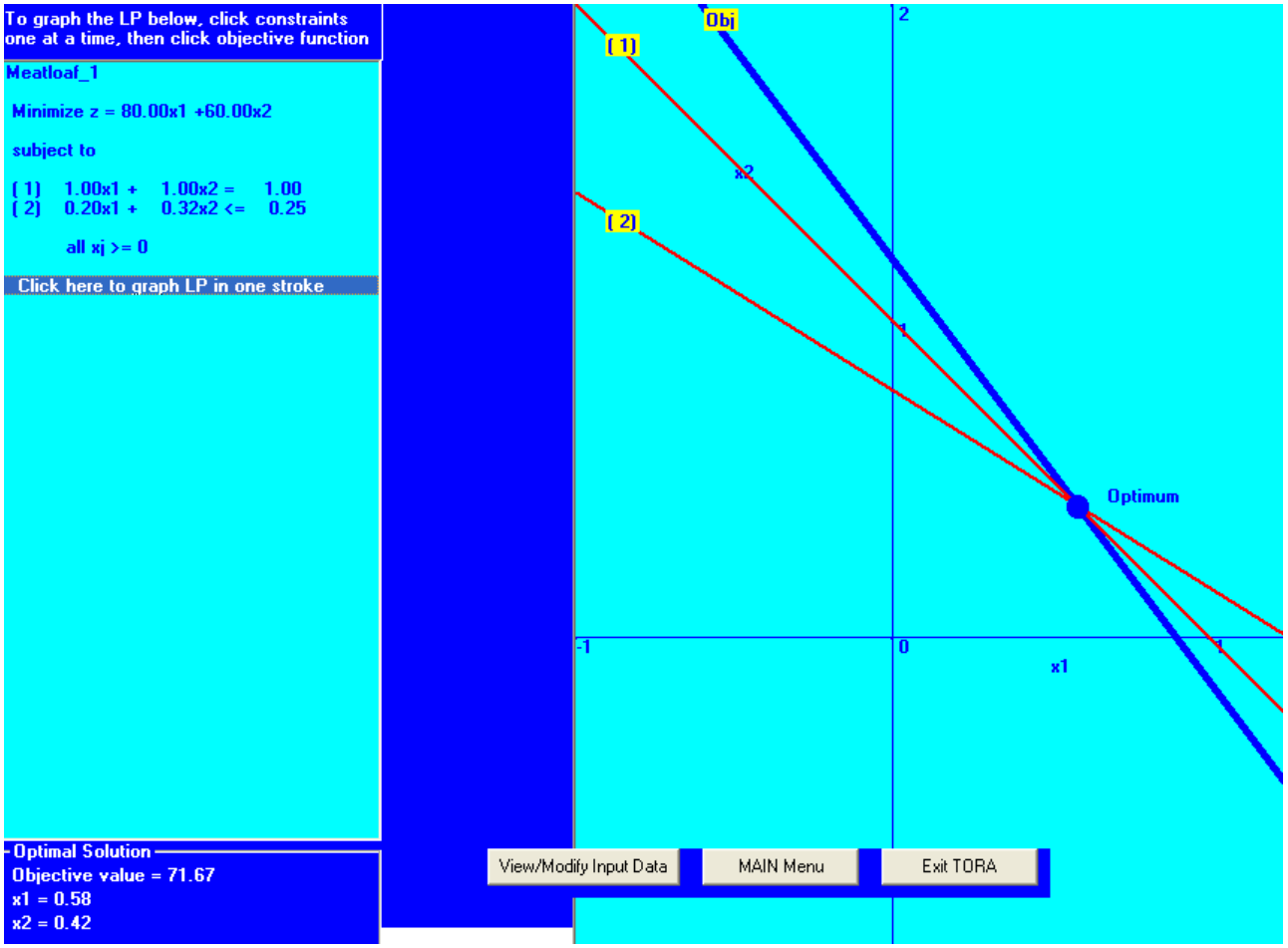

Figure 1: Optimum: .58 kilos beef, .42 kilos pork, cost 71.67 cents

Solution to second formulation (via TORA)

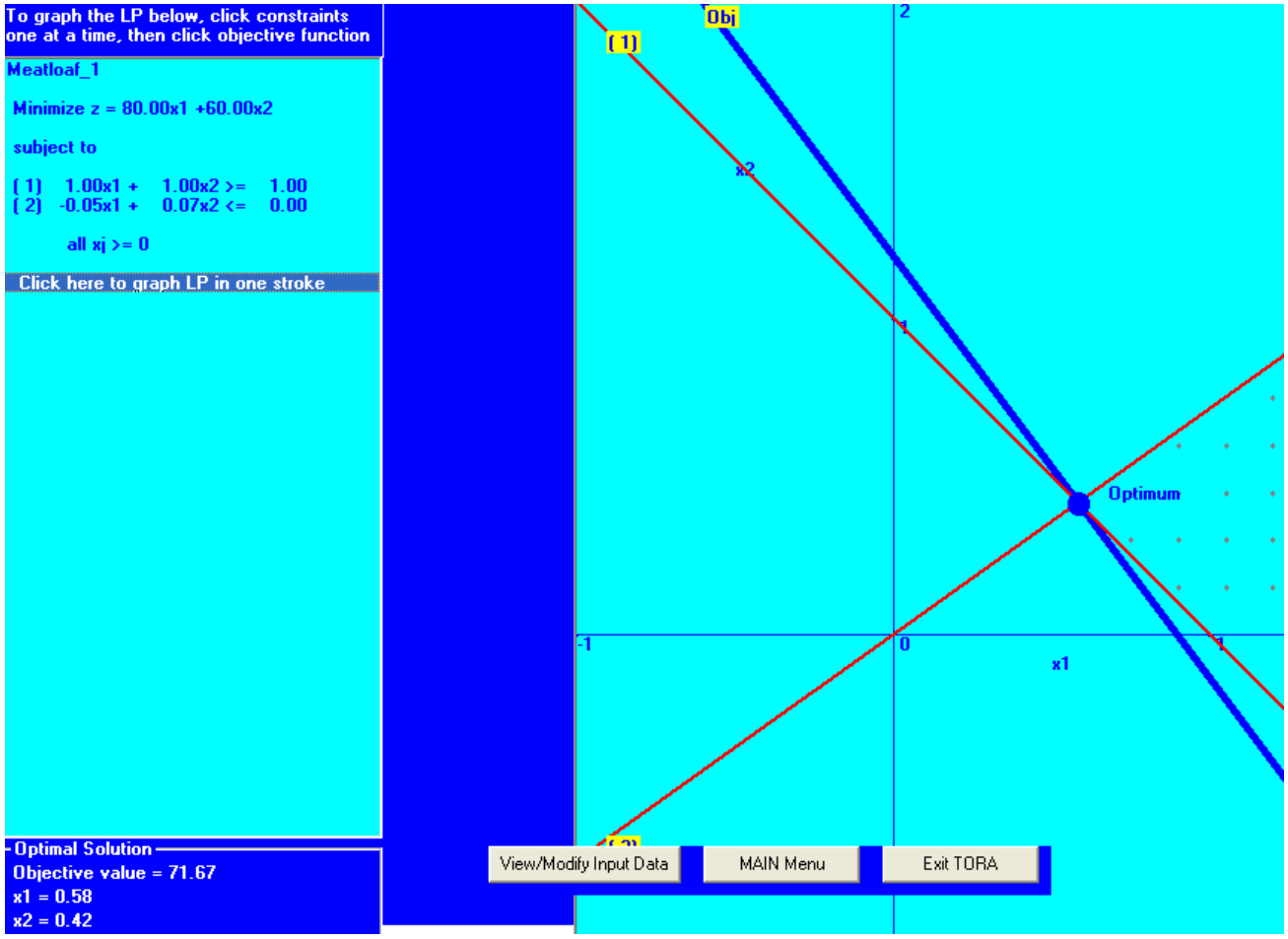

Figure 2: Optimum: .58 kilos beef, .42 kilos pork, cost 71.67 cents