Simplex method — summary

Problem: maximize a linear objective, subject to linear "$\leq$" constraints, all variables non-negative (minimizing X is equivalent to maximizing $-X$, so no loss in assuming max. problem)

Step 1: Convert to standard form

Step 2: Add new variable z, and add the new constraint $z-$ objective $= 0$

Step 3: Form the initial tableau

- first column to identify basic variables
- last column for constants on right-hand sides of constraints
- in between, one column for each variable (beginning with z)
- first row for labels
- remaining rows for constraints (beginning with objective)

Step 4: Identify initial basic feasible solution ("all-slack")

- slack variables as basic, "real" variables as non-basic
- label each constraint row by basic var. occurring once in that row

Step 5: If no variables in objective row with negative coefficients,
STOP. Current basic feasible solution is optimal, and optimal objective value is last entry (solution column) of objective row

Step 6: Choose an entering variable and pivot column

- one with most negative objective coefficient, and associated column
- if tie, choose leftmost

Step 7: Choose a departing variable and pivot row

- for each basic var., take ratio of entry in solution column and entry in pivot column; take var. and row of smallest non-negative value
- if tie, choose topmost

Step 8: Pivot on pivot entry (intersection of pivot row and pivot column)

- scale pivot row so pivot element is one
- add multiples of pivot row to other rows (including obj. row) so rest of pivot column is zero
- change label of pivot row to that of entering variable

Step 9: Go back to Step 5 with the new tableau