Simplex method — summary

**Problem**: maximize a linear objective, subject to linear “≤” constraints, all variables non-negative (minimizing $X$ is equivalent to maximizing $-X$, so no loss in assuming max. problem)

**Step 1**: Convert to standard form

**Step 2**: Add new variable $z$, and add the new constraint $z -$ objective $= 0$

**Step 3**: Form the initial tableau
- first column to identify basic variables
- last column for constants on right-hand sides of constraints
- in between, one column for each variable (beginning with $z$)
- first row for labels
- remaining rows for constraints (beginning with objective)

**Step 4**: Identify initial basic feasible solution (“all-slack”)
- slack variables as basic, “real” variables as non-basic
- label each constraint row by basic var. occurring once in that row
Step 5: If no variables in objective row with negative coefficients, STOP. Current basic feasible solution is optimal, and optimal objective value is last entry (solution column) of objective row

Step 6: Choose an entering variable and pivot column
- one with most negative objective coefficient, and associated column
- if tie, choose leftmost

Step 7: Choose a departing variable and pivot row
- for each basic var., take ratio of entry in solution column and entry in pivot column; take var. and row of smallest non-negative value
- if tie, choose topmost

Step 8: Pivot on pivot entry (intersection of pivot row and pivot column)
- scale pivot row so pivot element is one
- add multiples of pivot row to other rows (including obj. row) so rest of pivot column is zero
- change label of pivot row to that of entering variable

Step 9: Go back to Step 5 with the new tableau