16 Quiz 4 (Dec. 8) Name: SOLUTIONS

All questions relate to the matrix game whose payoff matrix is as shown:

		P_2			
		t_1	t_2	t_3	t_4
P_1	s_1	0	−1	1	−2
	s_2	−2	1	−1	−2
	s_3	1	−3	0	1

1. Say what is P_1's maximum security (pure) strategy (there might be more than one), what is P_2's maximum security (pure) strategy (again, there might be more than one), and what the values of u_1 and u_2 are.

row mins

$$\begin{array}{cccc|c} 0 & -1 & 1 & -2 & -2 \\ -2 & 1 & -1 & -2 & -2 \\ 1 & -3 & 0 & 1 & -3 \\ \hline 1 & 1 & 1 & 1 & \end{array}$$

col maxs

Max of row mins; P_1's max security strategies are s_1, s_2, $u_1 = -2$

Min of column maxes; P_2's max security strategies are t_1, t_2, t_3, t_4, $u_2 = 1$

2. Does the game have a saddle point? If so, identify it; if not, say briefly why it does not.

No. None of the row mins are column maxes, so there is no entry that is simultaneously a row min + column max.

[OR: $u_1 < u_2$, so apply Thm 9.3.2]

3. Suppose that P_1 plays the mixed strategy $(0, 4/7, 3/7)$. What is the worst-case (from P_1's perspective) payoff to P_1, over all possible (mixed or pure) responses from P_2?

$$\begin{bmatrix} 0 & \frac{4}{7} & \frac{3}{7} \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 & -2 \\ -2 & 1 & -1 & -2 \\ 1 & -3 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-5}{7} & \frac{-5}{7} & \frac{4}{7} & \frac{-5}{7} \end{bmatrix}}$$

Min here is $-\frac{5}{7}$

4. Suppose that P_2 plays the mixed strategy $(3/7, 3/7, 0, 1/7)$. What is the worst-case (from P_2's perspective) payoff to P_1, over all possible (mixed or pure) responses from P_1?

$$\begin{bmatrix} 0 & -1 & 1 & -2 \\ -2 & 1 & -1 & -2 \\ 1 & -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3}{7} \\ \frac{3}{7} \\ 0 \\ \frac{1}{7} \end{bmatrix} = \begin{bmatrix} \frac{-5}{7} \\ -\frac{5}{7} \\ -\frac{5}{7} \end{bmatrix}$$

Max here is $-\frac{5}{7}$

5. What can you say about the value of the game, based on your answers to the last two parts?

3) says $v_1 \geq -\frac{5}{7}$

4) says $v_2 \leq -\frac{5}{7}$

Since $v_1 = v_2$, can conclude that value of game is $-\frac{5}{7}$