6 Homework 4 (due Sept. 26) Name: SOLUTIONS

Note that the homework is due on **FRIDAY**! The purpose of this homework is to explore the tableau presentation of the simplex algorithm, and the method of artificial variables for generating an initial basic feasible solution. A separate part of this homework also explores Solver, the Excel LP tool.

General note on presenting solutions: The complete solution to an LP consists EITHER of the statement that the problem is unbounded, OR the statement that it has no feasible solution, OR the statement that it has a particular optimum value, achieved at a particular point. When I say "write the solution to the problem in the space provided", this is what I am asking for. If you use LP Assistant (which you will have to for some of these problems), you should take a screenshot of the LP Assistant tableaux and include it with the homework you turn in. PLEASE be careful to label your screenshot printouts in such a way that it is easy to match the printouts with the problems! Something like "Q 11 part (p)" written prominently on the screenshot printout will do.

Reading: Sections 3.5 and 3.6, and the handout on Solver.

1. Problem set 3.5, question 1 (page 89). For each part, reproduce the tableau and either circle all the entries on which is it valid to pivot, or say that the problem is completely resolved (with specifics: either the solution is unbounded, or ~~the~~ a particular optimum is achieved by a particular solution). DON'T use LP Assistant!

   - part (a)

basis	x_1	x_2	x_3	x_4	x_5	x_6	x_7	rhs
x_5	0	5	0	3	1	-1	8	39
x_3	0	6	1	-1	0	0	-6	10
x_1	1	9	0	8	0	-3	4	88
	0	6	0	-4	0	2	3	-75

(ii) applies, there is only one valid pivot point

- part (d)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0	5	0	−3	1	−1	8	3
x_3	0	6	1	1	0	0	−6	2
x_1	1	9	0	−8	0	−3	4	1
	0	−6	0	0	0	−2	3	$-75+z$

(i) applies. The x_6 column shows that the problem is unbounded from below

- part (e)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0	5	0	−3	1	1	8	60
x_3	0	6	1	−1	0	0	−6	30
x_1	1	9	0	−8	0	−3	7	50
	0	−6	0	0	0	−2	−3	$-75+z$

(ii) applies. There are three possible pivot points

- part (f)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0	−5	0	−3	1	−1	8	39
x_3	0	−6	1	−1	0	0	−6	0
x_1	1	9	0	−8	0	−3	4	88
	0	6	0	0	0	2	3	$-75+z$

(i) applies. The optimum value of the objective is 75, and one point where this is achieved is $x_1 = 88$, $x_5 = 39$, and all other $x_i = 0$.

2. Problem set 3.5, question 2 (page 90). Feel free to ignore the instructions, and use LP Assistant. Print out a screenshot of the LP Assistant solution, mark clearly which question it belongs to, and in the space below write the solution to the problem (optimum, point achieving optimum). Be careful getting the problems into canonical form!

- part (a)

For canonical form: add slack variables x_5, x_6, x_7, which can serve as basic variables in initial bfs. objective function is then already expressed in terms of non-basic variables.
For LP Assistant solution, see later.

- part (d)

For canonical form: use x_1, x_3, x_5 as basic variables (after multiplying 2^{nd} constraint by -1 → notice this makes rhs positive)
In terms of non-basic variables, obj is to maximize
$9x_2 - 2x_2 - 6x_6 + \cancel{20} - 2x_2 + x_4 - 4x_6 - \cancel{20} = 5x_2 + x_4 - 10x_6$;
so minimize $-5x_2 - x_4 + 10x_6$
For LP Assistant solution, see later

- part (f)

For canonical form, use x_1, x_2, x_3 as basic variables in initial bfs. Modify objective to

$3x_4 - 2x_5 + 5 - x_4$ or $2x_4 - 2x_5 + 5$;

so for initial tableau, $2x_4 - 2x_5 = z - 5$

For LP Assistant solution, see later.

Untitled Problem 1

Tableau

Mode
- Edit
- Pivot

Algorithm
- Simplex
- Dual Simplex

Display
- 1/2
- 1.0
- 1.00
- 1.000

Ratio

Basis	X1	X2	X3	X4	X5	X6	X7	RHS
x_5	8	-2	1	-1	1	0	0	50
x_6	3	5	0	2	0	1	0	150
x_7	1	-1	2	-4	0	0	1	100
	2	4	-4	7	0	0	0	0
x_3	8	-2	1	-1	1	0	0	50
x_6	3	5	0	2	0	1	0	150
x_7	-15	3	0	-2	-2	0	1	0
	34	-4	0	3	4	0	0	200
x_3	-2	0	1	$-\frac{7}{3}$	$-\frac{1}{3}$	0	$\frac{2}{3}$	50
x_6	28	0	0	$\frac{16}{3}$	$\frac{10}{3}$	1	$-\frac{5}{3}$	150
x_2	-5	1	0	$-\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	0
	14	0	0	$\frac{1}{3}$	$\frac{4}{3}$	0	$\frac{4}{3}$	200

H4, Q2, part a)

Optimum of -200 is reached

at $x_2 = 0$

$x_3 = 50$

$x_6 = 150$ (all other vars $= 0$)

Tableau

Mode

Edit

Pivot

Algorithm

Simplex

Dual Simplex

Display

1/2

1.0

1.00

1.000

Ratio

Basis	X1	X2	X3	X4	X5	X6	RHS
x_1	1	-3	0	-4	0	2	80
x_5	0	-2	0	1	1	4	20
x_3	0	1	1	0	0	3	10
	0	-5	0	-1	0	10	0
x_1	1	0	3	-4	0	11	90
x_5	0	0	2	1	1	10	40
x_2	0	1	1	0	0	3	10
	0	0	5	-1	0	25	50
x_1	1	0	11	0	4	51	250
x_4	0	0	2	1	1	10	40
x_2	0	1	1	0	0	3	10
	0	0	7	0	1	35	90

H4 Q2 part d),

Objective value of -90 is minimum,
(so for original problem, objective value of 90 is maximum), achieved at

$x_1 = 250$

$x_2 = 10$

$x_4 = 40$

all other vars $= 0$

Untitled Problem 2

Tableau

Mode
- Edit
- Pivot

Algorithm
- Simplex
- Dual Simplex

Display
- 1/2
- 1.0
- 1.00
- 1.000

Ratio

Basis	X1	X2	X3	X4	X5	RHS
x_1	1	0	0	-3	1	1
x_2	0	1	0	6	-5	6
x_3	0	0	1	-3	2	5
	0	0	0	2	-2	-5
x_5	1	0	0	-3	1	1
x_2	5	1	0	-9	0	11
x_3	-2	0	1	3	0	3
	2	0	0	-4	0	-3
x_5	-1	0	1	0	1	4
x_2	-1	1	3	0	0	20
x_4	$-\frac{2}{3}$	0	$\frac{1}{3}$	1	0	1
	$-\frac{2}{3}$	0	$\frac{4}{3}$	0	0	1

H4 Q2 part f)

x_1 column shows problem is unbounded from below.

3. Problem set 3.6, question 1 (page 99), part (a). DON'T use LP Assistant!!!!

Add artificial variables x_4, x_5, use simplex to solve:

Minimize w subject to

$$
\begin{aligned}
x_1 - x_2 \qquad\qquad + x_4 \qquad &= 1 \\
2x_1 + x_2 \quad - x_3 \qquad + x_5 &= 3 \\
x_4 + x_5 &= w
\end{aligned}
$$

Equivalent objective: $-3x_1 \quad + x_3 \qquad = w - 4$

Tableau:

basis	x_1	x_2	x_3	x_4	x_5	rhs
x_4	1	-1	0	1	0	1
x_5	2	1	-1	0	1	3
obj	-3	0	1	0	0	-4
x [illegible]	1	-1	0	1	0	1
x_5	0	3	-1	-2	1	1
	0	-3	1	3	0	-1
x_1	1		$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$1\frac{1}{3}$
x_2	0	1	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	0	0	0	1	1	0

Optimum w is reached at $x_1 = 1\frac{1}{3}$, $x_2 = \frac{1}{3}$ ($x_3 = x_4 = x_5 = 0$), optimum is 0. So, $x_1 = 1\frac{1}{3}$, $x_2 = \frac{1}{3}$ gives a feasible point of original system

4. Problem set 3.6, question 2 (page 99). For these three parts, please DO use LP Assistant. Print out a screenshot of the LP Assistant solution, and in the space below write the solution.

- part (a)

Optimum value of $-\frac{9}{5}$ is acheived at point $x_1 = 0$, $x_2 = \frac{23}{5}$, $x_3 = \frac{11}{5}$

- part (d)

The objective function is not bounded (it can be made arbitrarily large)

- part (f)

Optimum objective value of -135 is achieved at $x_1 = 0$
$x_2 = 0$
$x_3 = 9$
$x_4 = 21$

H4 , Q4 part (a)

Untitled Problem 3

Tableau

Mode
- Edit
- Pivot

Algorithm
- Simplex
- Dual Simplex

Display
- 1/2
- 1.0
- 1.00
- 1.000

Ratio

Basis	X1	X2	X3	X4	X5	RHS
x_4	3	2	-1	1	0	7
x_5	1	-1	3	0	1	2
	2	2	-5	0	0	0
	-4	-1	-2	0	0	-9
x_4	0	5	-10	1	-3	1
x_1	1	-1	3	0	1	2
	0	4	-11	0	-2	-4
	0	-5	10	0	4	-1
x_2	0	1	-2	$\frac{1}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$
x_1	1	0	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{11}{5}$
	0	0	-3	$-\frac{4}{5}$	$\frac{2}{5}$	$-\frac{24}{5}$
	0	0	0	1	1	0
x_2	2	1	0	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{23}{5}$
x_3	1	0	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{11}{5}$
	3	0	0	$-\frac{1}{5}$	$\frac{8}{5}$	$\frac{9}{5}$
	0	0	0	1	1	0

LP Assistant

Problem Useful Aids

Untitled Problem 4

Tableau

Mode: Edit, Pivot

Algorithm: Simplex, Dual Simplex

Display: 1/2, 1.0, 1.00, 1.000

Ratio

Basis	X1	X2	X3	X4	X5	X6	RHS
x_3	1	-1	1	0	0	0	3
x_4	2	-1	0	1	0	0	0
x_6	1	1	0	0	-1	1	12
	3	-1	0	0	0	0	0
	-1	-1	0	0	1	0	-12
x_3	0	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	3
x_1	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0
x_6	0	$\frac{3}{2}$	0	$-\frac{1}{2}$	-1	1	12
	0	$\frac{1}{2}$	0	$-\frac{3}{2}$	0	0	0
	0	$-\frac{3}{2}$	0	$\frac{1}{2}$	1	0	-12
x_3	0	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	7
x_1	1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	4
x_2	0	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	8
	0	0	0	$-\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	-4
	0	0	0	0	0	1	0
x_3	2	0	1	0	-1	1	15
x_4	3	0	0	1	-1	1	12
x_2	1	1	0	0	-1	1	12
	4	0	0	0	-1	1	12
	0	0	0	0	0	1	0

H4, Q4 part (d)

LP Assistant

Problem Useful Aids

Untitled Problem 5

Tableau

Mode
- Edit
- Pivot

Algorithm
- Simplex
- Dual Simplex

Display
- 1/2
- 1.0
- 1.00
- 1.000

Ratio

Basis	X1	X2	X3	X4	Y5	X6	RHS
x_5	1	1	-1	1	1	0	12
x_6	-2	3	0	2	0	1	42
	8	-2	-1	-6	0	0	0
	1	-4	1	-3	0	0	-54
x_2	1	1	-1	1	1	0	12
x_6	-5	0	3	-1	-3	1	6
	10	0	-3	-4	2	0	24
	5	0	-3	1	4	0	-6
x_4	1	1	-1	1	1	0	12
x_6	-4	1	2	0	-2	1	18
	14	4	-7	0	6	0	72
	4	-1	-2	0	3	0	-18
x_4	-1	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	21
x_3	-2	$\frac{1}{2}$	1	0	-1	$\frac{1}{2}$	9
	0	$\frac{15}{2}$	0	0	-1	$\frac{7}{2}$	135
	0	0	0	0	1	1	0

H4, Q4 part f)

5. **Problem set 3.6, question 3 (page 100) (Setup the problem in the space below. Then solve it using LP Assistant).**

x_1 = # lbs of A used in a 1 lb blend
x_2 = # " " B " " " " "
x_3 = # " " C " " " " "

Minimize $57x_1 + 13x_2 + 20x_3$

subject to

$$\begin{aligned} 25x_1 \quad\quad + 10x_3 - x_4 \quad &= 20 \\ 40x_1 + 30x_2 + 15x_3 \quad - x_5 &= 30 \\ x_1 + x_2 + x_3 \quad\quad &= 1 \end{aligned}$$

(x_4, x_5 are slack variables)

See later for LP Assistant tableau

Solution: Optimum cost is $\frac{134}{3}$ cents/lb,

achieved at $x_1 = \frac{2}{3}$

$x_2 = 0$

$x_3 = \frac{1}{3}$

Untitled Problem 6

Tableau

Mode: Edit, Pivot
Algorithm: Simplex, Dual Simplex
Display: 1/2, 1.0, 1.00, 1.000
Ratio

Basis	X1	X2	X3	X4	X5	X6	X7	X8	RHS
x_6	25	0	10	-1	0	1	0	0	20
x_7	40	30	15	0	-1	0	1	0	30
x_8	1	1	1	0	0	0	0	1	1
	57	13	20	0	0	0	0	0	0
	-66	-31	-26	1	1	0	0	0	-51
x_6	0	$-\frac{75}{4}$	$\frac{5}{8}$	-1	$\frac{5}{8}$	1	$-\frac{5}{8}$	0	$\frac{5}{4}$
x_1	1	$\frac{3}{4}$	$\frac{3}{8}$	0	$-\frac{1}{40}$	0	$\frac{1}{40}$	0	$\frac{3}{4}$
x_8	0	$\frac{1}{4}$	$\frac{5}{8}$	0	$\frac{1}{40}$	0	$-\frac{1}{40}$	1	$\frac{1}{4}$
	0	$-\frac{119}{4}$	$-\frac{11}{8}$	0	$\frac{57}{40}$	0	$-\frac{57}{40}$	0	$-\frac{171}{4}$
	0	$\frac{27}{2}$	$-\frac{5}{4}$	1	$-\frac{13}{20}$	0	$\frac{33}{20}$	0	$-\frac{3}{2}$
x_6	0	-19	0	-1	$\frac{3}{5}$	1	$-\frac{3}{5}$	-1	1
x_1	1	$\frac{3}{5}$	0	0	$-\frac{1}{25}$	0	$\frac{1}{25}$	$-\frac{3}{5}$	$\frac{3}{5}$
x_3	0	$\frac{2}{5}$	1	0	$\frac{1}{25}$	0	$-\frac{1}{25}$	$\frac{8}{5}$	$\frac{2}{5}$
	0	$-\frac{148}{5}$	0	0	$\frac{37}{25}$	0	$-\frac{37}{25}$	$\frac{11}{5}$	$-\frac{211}{5}$
	0	19	0	1	$-\frac{3}{5}$	0	$\frac{8}{5}$	2	-1
x_5	0	$-\frac{95}{3}$	0	$-\frac{5}{3}$	1	$\frac{5}{3}$	-1	$-\frac{5}{3}$	$\frac{5}{3}$
x_1	1	$-\frac{2}{3}$	0	$-\frac{1}{15}$	0	$\frac{1}{15}$	0	$-\frac{2}{3}$	$\frac{2}{3}$
x_3	0	$\frac{5}{3}$	1	$\frac{1}{15}$	0	$-\frac{1}{15}$	0	$\frac{5}{3}$	$\frac{1}{3}$
	0	$\frac{53}{3}$	0	$\frac{37}{15}$	0	$-\frac{37}{15}$	0	$\frac{14}{3}$	$-\frac{134}{3}$
	0	0	0	0	0	1	1	1	0

H4, Q5

6. **Problem set 3.6, question 5 (page 101)**

We can only learn of unboundedness using the unboundedness criterion (thm 3.4.2), which, since we are trying to conclude unboundedness for the original problem, must be applied to the original tableau. But the unboundness criterion can only be applied to the original tableau when the original tableau is in canonical form (with a basic feasible solution using only original variables. If the first (artificial) stage of the simplex method has not yet ended, there are still artificial variables in the basis, and the original tableau is not property in canonical form, so unboundedness criterion cannot yet be applied.