8 Quiz 2 (Oct. 1) Name: SOLUTIONS

1. The tableau below reveals that the linear programming currently being solved is not bounded from below.

Basis	x_1	x_2	x_3	x_4	x_5	x_6	rhs
x_3	2	0	1	0	0	2	12
x_4	-4	-3	0	1	0	-1	6
x_5	3	-5	0	0	1	3	20
Obj	5	-2	0	0	0	-2	40

- Part (a): Suppose that x_2 enters the set of basic variables, with some value X, while all the other non-basic variables (x_1, x_6) remain at value 0. What value (it will depend on X) should the basic variable x_4 be set to, to maintain feasibility?

With x_1, x_6 set to 0, Constraint # 2 says
$-3x_2 + x_4 = 6$, or $x_4 = 6 + 3x_2$
Setting $x_2 = X$ forces $x_4 = \underline{6+3X}$ to maintain feasibility

- Part (b): Again, Suppose that x_2 enters the set of basic variables, with some value X, while all the other non-basic variables remain at value 0. What will be the new value of the objective function? (It will depend on X.)

With x_1, x_6 set to 0, current objective value satisfies
$-2x_2 = z + 40$ or $z = -40 - 2x_2$
Setting $x_2 = X$ cause $z = \underline{-40 - 2X}$

- Part (c): Give an example of a feasible point for this problem that has objective value -240.

Want $z = -40 - 2X = -240$, so $X = 100$
Keeping $x_1, x_6 = 0$, moving x_2 to 100 moves x_4 to 306
Since x_2 has coefficient 0 in constraint with x_3 basic, x_3 can stay at 12. Constraint #3 says
$-5x_2 + x_5 = 20$ or $x_5 = 20 + 5x_2$, so x_5 moves to 520.

Point $\underline{(0, 100, 12, 306, 520, 0)}$ has objective value -240.

2. Consider the following LP problem: Minimize $a + b + c$ subject to

$$\begin{aligned} 3a - 2b - c &\le 2 \\ -a + 4b + c &\ge 4 \\ 2a + 3b - 2c &= 4 \end{aligned}$$

as well as $a, b, c \ge 0$.

Slack variables

artificial variables

- Set up the initial simplex tableau for this problem, including all necessary slack variables and artificial variables. If using artificial variables, be sure to correctly present the artificial objective function.

basic	a	b	c	d	e	f	g	rhs
d	3	−2	−1	1	0	0	0	2
f	−1	4	1	0	−1	1	0	4
g	2	3	−2	0	0	0	1	4
obj	1	1	1	0	0	0	0	0
Artifical obj	−1	−7	1	0	1	0	0	−8

Artificial objective is $w = f + g = 8 - a - 7b + c + e$

$f = 4 + a - 4b - c + e$

$g = 4 - 2a - 3b + 2c$

so $-a - 7b + c + e = w - 8$

- Say which of the following three things will happen when you begin running the simplex algorithm on your tableau:
  - **A**: The algorithm immediately terminates with "optimum reached" (in this case, say what the optimum is, and what point it is reached at)
  - **B**: The algorithm terminates with "the problem is not bounded from below" (in this case, circle the entries of the tableau that allow you to conclude this)
  - **C**: A pivoting occurs (in this case, circle all entries in the tableau on which it would be legitimate to pivot)

There are two possible places where a pivoting can occur.