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Math 30-440: Probability and Statistics
Spring Semester 2008
Final Exam
May 5

This examination contains 8 problems on 10 sheets of paper (including the front cover). You can use your own textbook and notes. Do all your work on the paper provided and show all your computations. You are taking this test under the honor code.

Scores

Question	Possible	Actual
1	18	
2	20	
3	22	
4	18	
5	20	
6	18	
7	16	
8	18	
Total	150	

GOOD LUCK !!!

1. In a certain system, the response Y is believed to be a linear function of the input x, with some normal error. That is,

$$Y = \alpha + \beta x + e$$

where α and β are constants, and e is a normal random variable with mean 0 and variance σ^2. Given the data

x_i	Y_i
-2	0
-1	0
0	1
1	1
2	3

find the least squares estimates for α and β.

2. Achievement test scores of all high school seniors in a certain state have mean 60 and variance 64. A random sample of 100 students from one large high school had a mean score of 58. Is there evidence to suggest that this high school is inferior? State any assumptions that you make.

3. **a)** If $X_1, \ldots, X_n$ is a sample from a distribution given by

$$f(x) = \begin{cases} \frac{4x^3}{\theta} e^{-\frac{x^4}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases},$$

find the maximum likelihood estimator of θ.

b) The following numbers are drawn independently from a uniform distribution on the interval $-\theta \leq x \leq \theta$:

$$-2, 3, 1, -1, -2.$$

Find the maximum likelihood estimator of θ.

4. A manufacturer of gunpowder has developed a new powder, which was tested in eight shells. The resulting muzzle velocities, in feet per second, were as follows:

3005	2990	3010	2990
3000	2995	2979	2991

Find a 95% confidence interval for the true average velocity μ for shells of this type. State any assumptions that you make.

5. A curious professor wants to estimate the proportion of students who favor partial credit exams over multiple-choice exams. Among 100 students currently in her class, 56 prefer partial credit exams.

a) Find a 90% confidence interval for the percentage of students who prefer *multiple-choice* exams.

b) If the estimate above is to be within 0.05 of the true percentage favoring multiple-choice exams (with a 90% confidence), how many students should be sampled?

6. In a study of the amount of calcium in drinking water undertaken as part of water-quality assessment, the same standard was run through the laboratory six times at random intervals. The six readings (in parts per million) were

$$9.5 \quad 9.6 \quad 9.3 \quad 9.5 \quad 9.7 \quad 9.4$$

Estimate σ^2, the variance for readings on this standard, using 90% confidence interval.

7. **a)** A certain null hypothesis H_0 is tested against an alternative H_1, and accepted at 5% significance. With the same data, will H_0 be accepted at 2% significance?

b) Suppose the null hypothesis H_0 is tested against the alternative H_1, and rejected at 5% significance. With the same data, will H_0 be rejected at 2% significance?

d) John constructs a $100(1-\alpha)\%$ two-sided confidence interval for the mean of a normal population based on a sample of size 100. Jill constructs a $100(1-\alpha)\%$ confidence interval based on a sample of size 400. How are the lengths of the two intervals related?

d) What is the interpretation of $\beta = 0$ in a linear regression?

8. Professor X. believes that students who skip Friday afternoon classes don't perform as well on exams as those who show up diligently. His students (especially those who like to start the weekend watching a Friday afternoon baseball game) are skeptical, and ask for proof. So Prof. X points out that on the last midterm, the scores of 20 randomly selected students who always attend class on Friday had a mean of 78 and a sample variance of 40, while the scores of 20 randomly selected students who never attend class on Friday had a mean of 74 and a sample variance of 40. Has the Prof. proved his point?

Be sure to state what your null and alternative hypotheses are, and at what significance level you are drawing your conclusion.