The combinatorial expression \( \binom{n}{r} \) is defined by
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!},
\]
but it also has a combinatorial interpretation, namely
\[
\binom{n}{r} \text{ counts the number of subsets of size } r \text{ of a set of size } n.
\]

As a result, there are often two ways to verify an equality involving \( \binom{n}{r} \): a direct verification, using the definition, and what is called a combinatorial argument, which is an argument that shows that both sides of the equality can be interpreted as counting the same thing.

Here’s an example. Consider the equality
\[
r \binom{n}{r} = n \binom{n-1}{r-1}.
\]
This can be verified directly from the definition:
\[
r \binom{n}{r} = r \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r)!} = n \frac{(n-1)!}{(r-1)!(n-r)!} = n \binom{n-1}{r-1}.
\]
But there is also a combinatorial argument that proves the identity. Imagine having to choose a committee of size \( r \) from a group \( n \) people, with the additional rule that one of the \( r \) people on the committee must be selected as committee chair. How many ways are there to select the committee-with-chair? One way to answer this question is to say that there are \( \binom{n}{r} \) ways to choose the committee, and then, once it has been chosen, there are \( r \) ways to choose the chair. By the basic principle of counting, this means that
\[
\text{Number of possible committees-with-chair} = r \binom{n}{r}.
\]
But here’s another way to answer the question. There has to be a chair, so we first choose who that is to be ($n$ choices). The remaining $r - 1$ members of the committee now have to be chosen from among the remaining $n - 1$ people in the group ($\binom{n-1}{r-1}$ choices). It follows that

$$ \text{Number of possible committees-with-chair} = n \binom{n-1}{r-1}. $$

Since both $r \binom{n}{r}$ and $n \binom{n-1}{r-1}$ count the same thing, they must be equal.

The second parts of Ross, Chapter 3, Problems 16 and 17, should be answered in a similar way to the argument above.