Probability and Statistics, Spring 2009

Practice II

Problem 1.

Let X and Y be continuous independent random variables, Y is uniformly distributed in $[0,1]$ and X has an exponential distribution with a parameter $\lambda = 1$.

Find $P(X + Y \geq 1)$.

Problem 2.

One-tenth of 1% of a certain type of RAM chip are defective. A student needs 50 chips for a certain board.

a) What is the probability that if she buys 53 chips, she'll have enough working chips for the board?

b) What is the smallest number of chips that she should buy in order for there to be at least a 99% chance of having at least 50 working chips?

Problem 3.

If X is an exponential random variable with a parameter $\lambda = 2$, find the CDF $F_Y(y)$ of $Y = 2X - 1$.

Problem 4.

A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are described by a normal random variable with standard deviation $\sigma = 0.3$ ounce, find a value of μ such that 8-ounce cups will overflow only 1% of the time.

Problem 5.

The distribution of resistance for resistors of a certain type is known to be normal. 9.85 % of all resistors have a resistance exceeding 10.257 Ohms, and 5.05 % have resistance smaller than 9.671 Ohms. What are the mean value and variation of the resistance distribution?

Problem 6.

Let X be a Poisson random variable with a parameter $\lambda = 2$. Find the expected value and the variance of $Y = 3^X$.

Problem 7.

Assume that the time of arrival of birds at a particular place on a migratory route, as measured in days from the first of the year (January 1 is the first day), is approximated as a normal random variable X with $\mu = 200$ and $\sigma = 20$ days.

a) What is the probability the birds will arrive after 200th day?

b) What is the probability the birds arrive after 160 days but on or before 210th day?

Problem 8.

If X is a random variable uniformly distributed over the interval $(0, 2)$, find $E(X^3 - X)$.

Problem 9.

The number of cars which stop at a gas station is modeled by a Poisson random variable with an average of 10 cars per hour. What is the probability that at most one car will stop at this gas station during the next 30 minutes?

Problem 10.

If X has an exponential density and $E(X) = 20$, find $P(X > 20)$.

Problem 11.

One microgram of radium contains 10^{16} atoms. The probability that a single atom will disintegrate during a one-millisecond time is $p = 10^{-15}$. Approximate the probability that more than two atoms will disintegrate in one millisecond.