These problems are provided as practice for the sort of material covered on the exam. It is not necessarily indicative of the length or difficulty of the exam questions.

1. At the end of the day in the amusement park your friend wants try out the white water rafting ride while you want to go on the popular new roller coaster. Suppose that the number of minutes it takes to go on the white water ride is distributed $\mathcal{N}(30, 9)$ while the time it takes to ride the new roller coaster has the distribution $\mathcal{N}(50, 16)$. If the park closes in 1.5 hours what's the probability you will get to go on both rides? Assume it takes no time to go from one to the other.

2. During the slack time from 1pm to 5pm, customers arrive at Reckers at a constant rate of 11 per hour. A customer has just arrived. Let T be the time until the next customer arrives.

   (a) Using an exponential random variable to model T, calculate the probability that it will be more than 10 minutes until the next customer arrives.

   (b) Suppose that 5 minutes have passed, and no customer has arrived. Calculate the probability that it will be more than a further 10 minutes until the next customer arrives.

3. Compute the expectation and variance of a random variable with density function:

$$f(x) = \begin{cases} \frac{1}{3x^4} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

4. A high school health teacher knows that the body weight of typical students is normally distributed with mean 160 lbs and standard deviation of 80. He asks the school nurse to weigh the 16 students in his class, to see if he can show up the other health teachers by rejecting at .05 significance the hypothesis that his influence has no effect on student weight. After adding up the total weight of all his students he is devastated to see that it is 2100 pounds. On investigation he discovers that his dastardly rival has bribed the nurse to report that two randomly selected students of his having 1.5 times their true weight (and to report all the others correctly). Summer vacation prevents him from weighing the students again.

   a Can you help him undo the damage by finding a distribution for the reported weight? Hint: does it matter which two students if they are chosen randomly?

   b Using this corrected distribution does our hero succeed in rejecting the null hypothesis with significance .05?

5. Your boss needs to know the average compressive force (in pounds per square inch) the type of concrete you are using for a new bridge will support before failing. Using the company labratory you test 10 samples and discover they fail at the following pressures:

$$3000,\ 3300,\ 3180,\ 3360,\ 3240,\ 3420,\ 3120,\ 3600,\ 3300,\ 3480$$

Assuming the pressure at which a test pieces fail is normally distributed give your boss a 95% confidence interval for the mean pressure this kind of concrete can withstand.

6. Your apartment mate and you have a rule that whoever takes the last lightbulb from the closet must purchase the next package. Suppose you both replace the (single) bulb in your respective rooms today leaving only one bulb in the closet but your apartment mate has a second replacement bulb hidden in his room. Compute the probability you have to buy the next package conditional on your roommate resorting to his backup before your lightbulb burns out. You may assume that the lifetime of these light bulbs is exponentially distributed with a mean of 6 months.

7. Give short answers to each of the following questions.

   (a) What is a Type I error in a hypothesis test?

   (b) What is a Type II error in a hypothesis test?

   (c) A certain null hypothesis is accepted at 2% significance. With the same data, will it be accepted at 5% significance?

   (d) John constructs a 90% confidence interval for a certain parameter. Mary uses the same data to construct a 95% confidence interval for the same parameter. Whose interval is shorter?

8. I want to test a hypothesis about the mean μ of a certain normal population whose variance is known to be 4. My null hypothesis is $H_0 : \mu = 16$ and my alternative is $H_1 : \mu \neq 16$. Initially I sample from the population 25 times.

   (a) If I'm testing at 5% significance, what is the range of values of $\bar{X}$, my sample mean, that will lead to me accepting the null?

   (b) Suppose that the true mean is actually 20. What is the probability that I will incorrectly accept the null?

   (c) How large a sample would I have to take to make sure that there is only 5% probability of incorrectly accepting the null when the true mean is actually 20?

9. I'm willing to use a certain manufacturer's theodolite (angle-measuring device) as long as I am sure that it is consistent in its readings. So I test the theodolite by measuring the same angle 20 times; the answers I get have sample variance .8 degrees. I conclude that I am 90% confident that the variance σ^2 of measurements for the theodolite is no more than c.

   (a) What is a reasonable assumption to make about the distribution of measurements for this theodolite that would allow me to compute c?

   (b) What is c?

10. If $X_1, \ldots, X_n$ is a sample from a distribution given by

$$f(x) = \begin{cases} \frac{4x^3}{\theta} e^{-\frac{x^4}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases},$$

find the maximum likelihood estimator of θ.

11. The following numbers are drawn independently from a uniform distribution on the interval $-\theta \leq x \leq \theta$:

$$-2, 3, 1, -1, -2.$$

Find the maximum likelihood estimator of θ.

12. Experience has shown that when I am writing reports in the morning, I tend to make on average 1 typo per page. When I write in the afternoon, I make on average a .5 typos per page (1 per two pages). On report-writing days, I write four pages in the morning and five in the afternoon. Using a Poisson distribution (or distributions) to model the number of typos I make on report-writing days, answer the following questions:

    (a) Compute the probability that I make 2 or fewer typos in the morning.

    (b) What is the probability that I make 2 or fewer typos during the whole day?

13. When a stabilizer is used on a poles being dropped into the ocean to act as foundations for an oil rig, the x and y coordinates of the distance from the final location of the pole to its intended location are independent and normally distributed, each with mean 0 and standard deviation σ meters. The value of σ can be set in advance, but the smaller σ is the costlier the process.

    (a) If σ is set to 3, what is the probability that the pole will end up within 1 meter of its intended destination?

    (b) what should σ be set to so that one can be 99% certain that the pole will end up within 1 meter of its intended destination?

14. The temperature of a steel rod four hours after tempering is known to be normally distributed with mean 75 degree Celsius and standard deviation 25.

    (a) Compute the probability that a rod has temperature above 100 degrees Celsius four hours after tempering.

    (b) I can begin using a rod after tempering once its temperature has dropped below 100 degrees Celsius. If 8 rods are set aside for four hours after tempering, what's the probability that I can use at least 6 of them at this time?

15. A container ship carrying 900 containers of low quality iron ingots and 100 medium quality iron ingots capsizes in the storm and the next morning a large container of iron ingots is found on the beach. The percent impurity in a medium quality ingot is distributed $\mathcal{N}(1, .16)$ while the percent impurity of the low quality ingot is distributed $\mathcal{N}(2.1, 1)$.

   (a) Before any examination of the container is done what is the probability it contains medium quality ingots?

   (b) If 4 ingots from the container are tested and the average percent impurity found to be 1.4 what probability should you now assign to the container carrying medium quality ingots?

16. Your coworker seems to have been suspiciously successful in investing his money. Since investing his entire inheritance in a single company 10 years ago it has appreciated to $R = 2.5$ times its initial value. You suspect him of having had insider information when he invested. Financial theory suggests that, investing without any special knowledge, the natural log of your appreciation R after T years should have the following distribution $\ln R \backsim \mathcal{N}(.05T, .01T)$. Assuming it would be reasonable to model the effect of insider information as an increased expectation for $\ln R$ can you, with confidence .05, refute the hypothesis that your coworker did nothing wrong?