Math 30440 — Probability and Statistics

Spring 2010 second mid-term exam, March 22 2010

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Name: SOLUTIONS

This examination contains 5 problems on 7 pages (including the front cover and a standard normal table at the back). It is closed-book. You may use up to 2 pages of handwritten notes. You may use a calculator, but only for arithmetic; you must calculate all integrals by hand. **Show all your work** on the paper provided. The honor code is in effect for this examination.

Scores

Question	Score	Out of
1		10
2		10
3		10
4		10
5		10
Total		70

GOOD LUCK !!!

1. Data suggests that a newly purchased netbook will have a lifetime of 9 months before it experiences its first breakdown.

   (a) Using an appropriate random variable to model the lifetime of a netbook, calculate the probability that the first breakdown will occur sometime later than one year after purchase.

Units = months

Average # breakdowns per month $= \frac{1}{9}$

Use exponential $\lambda = \frac{1}{9}$ to model lifetime X

$$P(X > 12) = \int_{12}^{\infty} \frac{1}{9} e^{-\frac{1}{9}x} \, dx$$

$$= \left[-e^{-\frac{1}{9}x} \right]_{12}^{\infty}$$

$$= e^{-\frac{4}{3}}$$

   (b) Your new netbook has been working without a breakdown for 6 months. Calculate the probability that it will continue working without a breakdown for at least another year.

By memorylessness,

$$P(X > 18 \mid X > 6) = P(X > 12)$$

$$= e^{-\frac{4}{3}}$$

2. An online store is selling 120 umbrellas produced by the firm `MyUmbrella.com`. 80% of this batch of umbrellas are new models. Miss Vicky is an avid umbrella collector. She also likes surprises and therefore she purchases 5 umbrellas chosen randomly from the 120. What is the probability that exactly 3 of the 5 she purchases are new models?

X = # of models

= hypergeometric with $N = .8 \times 120 = 96$

$M = 24$

$n = 5$

$$P(X=3) = \frac{\binom{96}{3}\binom{24}{2}}{\binom{120}{5}} = .2069\ldots$$

[Note: Since she chooses <u>5</u> umbrellas

(rather than randomly choosing from the 120 umbrellas 5 times)

X is hypergeometric, <u>not</u> binomial]

3. A total of 20,000 transistors were produced by a factory in a given day. Each transistor is defective with probability 10^{-4}, and defects are independent of each other. The factory foreman wants to know the probability that a container of transistors containing the total production for the day will contain fewer than 3 defective transistors.

(a) Write down an expression which gives the exact probability that the foreman wants to compute (no need to evaluate the expression).

X = # defectives

= Binomial (n, p) where $n = 20000$, $p = 10^{-4} = .0001$

$$P(X < 3) = \sum_{k=0}^{2} \binom{20000}{k} (10^{-4})^k (1-10^{-4})^{20000-k}$$

(b) Using an appropriate Poisson random variable, approximate the probability that the foreman wants to compute (for this part you should do the calculation).

For n large, p small

Binomial $(n, p) \approx$ Poisson (λ), $\lambda = np$

Here $\lambda = \frac{20000}{10000} = 2$

$$P(X<3) \approx \sum_{k=0}^{2} \frac{2^k}{k!} e^{-2}$$

$$= .677\ldots$$

4. People's reaction to a certain stimulus is known to be uniformly distributed between .2 of a second and 1.6 seconds.

(a) What is the mean and variance of the response time?

$R = \text{Uniform}\,[.2, 1.6]$

$\mu = \frac{1.8}{2} = .9$

$\sigma^2 = \frac{(1.6 - .2)^2}{12} = .16\dot{3}$

(b) What is the probability that a randomly chosen subject will take longer that 1 second to react to the stimulus?

$P(R > 1) = \frac{1.6 - 1}{1.6 - .2}$

$= \frac{.6}{1.4}$

$= .428\ldots$

(c) If 30 subjects are selected at random, what is the approximate probability that their average response times is greater than 1 second?

$\bar{R} = \text{Average} \approx \text{Normal}\left(.9, \frac{.16\dot{3}}{30}\right)$

$= \text{Normal}(.9, .005\dot{4})$

$P(\bar{R} > 1) = P\left(Z > \frac{1 - .9}{\sqrt{.005\dot{4}}}\right)$

$= P(Z > 1.355\ldots)$

$= .0869$

5. One hour after a rod is taken out of an annealing chamber, its temperature is normally distributed with mean 136 and variance 64. To be tempered, the rod should have a temperature below 124.

(a) Calculate the probability that a rod is ready for tempering one hour after being removed from the annealing chamber.

$$P(X < 124) = P\left(Z < \frac{124-136}{\sqrt{64}}\right)$$

$$= P(Z < -1.5)$$

$$= .0668$$

(X = temperature)

(b) Suppose 400 rods come out of the annealing chamber at once. Assuming that the temperatures of the rods after one hour are independent, use the Central Limit Theorem to estimate the probability that at least 35 of the rods are ready for tempering after one hour.

X = # of rods with temp < 124

X = Binomial $(400, .0668)$

$\approx$ Normal $(26.72, 24.94)$

So $$P(X > 35) \approx P\left(Z > \frac{35-26.72}{\sqrt{24.94}}\right)$$

$$= P(Z > 1.66)$$

$$= .0485$$

TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{x} e^{-y^2/2}\, dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998