MATH 30440 – SPRING 2010 – HOMEWORK 10 SOLS

8.3) a) $$\frac{\bar{X} - \mu_0}{\frac{20}{\sqrt{n}}} = \frac{52.5 - 50}{\frac{20}{\sqrt{64}}} = 1$$

reading from standard normal if null hypothesis is true.

$$\text{p-value} = P(Z > 1) + P(Z < -1)$$

b) $$\frac{55 - 50}{\frac{20}{\sqrt{64}}} = 2$$

$$\text{p-value} = P(Z > 2) + P(Z < -2)$$

c) $$\frac{57.5 - 50}{\frac{20}{\sqrt{64}}} = 3$$

$$\text{p-value} = P(Z > 3) + P(Z < -3)$$

8.4) $\bar{X} = 8.179$, $n = 10$

$H_0 : \mu = 8.2$

$H_1 : \mu \neq 8.2$

If null is true, $\dfrac{\bar{X} - 8.2}{\frac{.02}{\sqrt{10}}} = -3.3$

is reading from standard normal

p-value $= P(Z > 3.3)$ or $P(Z < -3.3)$

$\approx .001$

So: reject null at 10% significance

" " " 5% "

8.5) We are asked to see if their is evidence for unacceptability, so take this as alternative; take "acceptable" as null

$H_0 : \mu = 200$ (or $\mu \geq 200$)

$H_1 : \mu < 200$.

Test: Compute $\dfrac{\bar{X} - 200}{5/\sqrt{8}} = TS$

5% significance test:

If $TS \leq -1.645$, reject null

10% significance test:

If $TS \leq -1.28$, reject null

8.7) a) Two-sided hypothesis test of mean of normal population, variance known

b) Use $$n \approx \frac{\sigma^2 (z_{\alpha/2} + z_\beta)^2}{(\mu_1 - \mu_0)^2}$$

with
$$\mu_1 - \mu_0 = .03$$
$$\alpha = \beta = .05$$
$$\sigma = .02$$

to get $\underline{n = 6}$

c) (strongly) reject null

d) Extremely high

8.7) c) (elaborating)

$\bar{X} = 8.31$, so $\dfrac{\bar{X} - 8.2}{\frac{.02}{\sqrt{6}}} = 13.47$

So p-value $= 2P(Z > 13.47) \approx 0$;

reject null at any reasonable significance

d) (elaborating)

$\beta(8.32) = P(\text{accepting } H_0 \mid \mu = 8.32)$

$= P\left(8.2 - 1.96\frac{.02}{\sqrt{6}} \le \bar{X} \le 8.2 + 1.96\frac{.02}{\sqrt{6}} \;\middle|\; \bar{X} = \text{Normal}\left(8.32, \left(\frac{.02}{6}\right)\right)\right.$

$= P(8.184 \le \bar{X} \le 8.216 \mid \bar{X} = \text{Normal}(8.32, .000066\ldots$

$= 0$ (after some standardization)

So $P(\text{rejecting } H_0 \mid \mu = 8.32) \approx 1$.

8.9) μ = mean time to absorption

$H_0 : \mu \geq 10$

$H_1 : \mu < 10$ ← this is what Glaxo is claiming; it should not be accepted unless there is evidence in favour of it, so we take this as alternative

8.11) a) $\dfrac{\bar{X} - 100}{\frac{5}{\sqrt{20}}} = 4.47$

p-value $= P(Z \geq 4.47) \simeq 0$

b) $\dfrac{\bar{X} - 100}{\frac{10}{\sqrt{20}}} = 2.23\ldots$

p-value $= P(Z \geq 2.23)$

c) $\dfrac{\bar{X} - 100}{\frac{15}{\sqrt{20}}} = 1.49$

p-value $= P(Z \geq 1.49)$

8.12) $H_0 : \mu = 3$

$H_1 : \mu < 3$

$\bar{X} = 2.95, \quad n = 2500, \quad \sigma = 1$

a) $$\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = -2.5$$

p-value $= P(Z < -2.5) \rightarrow$ very small, so reject null at 5%

b) Data gives evidence that new toothpaste performs better, but a drop from 3 to 2.95 is not all that great ...

8.14) $H_0 : \mu = 24$ μ = average per minute

$H_1 : \mu \neq 24$

Data : $\bar{X} = 22.5$

$S = 3.1$

Test : Compare $\left| \dfrac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} \right|$ to $t_{n-1, .025}$

Here $\dfrac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = -2.9$, and $t_{35, .025} = 1.96$

So reject null at 5% significance.

8.15) $H_0 : \mu = .8$ $\bar{X} = 1$, $S = .3$, $n = 28$

$H_1 : \mu \neq .8$

$\dfrac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = 3.52$

Since $3.52 > t_{27, .025}$,

reject null at 5%. (We can conclude, at 5% significance, that alcohol affects mean response time)

8.17) $H_0 : \mu = 98.6$

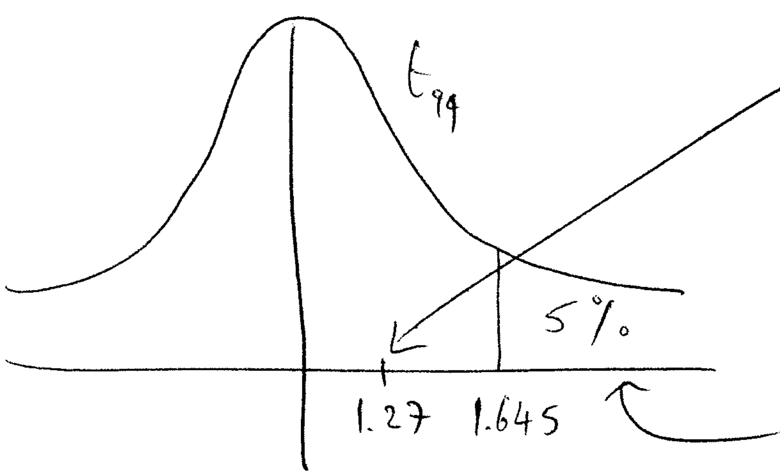
$H_1 : \mu > 98.6$ [To prove claim, we need to see evidence in it's favour, so we put claim as alternative hypothesis]

Data: $\bar{X} = 98.74$

$S = 1.1$

$n = 100$

$$\frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = 1.27$$



accept null with this reading

need to see a reading in here to see enough evidence to reject null at 5% significance

+ Since we accept null at 5%, we also accept null at 1%.

8.21) $H_0 : \mu \geq 240$

$H_1 : \mu < 240$ ← We're asked if there is evidence in favour of claim "specifications not being met". So take this as null.

Data : Compute $\bar{X}$, S from data.

Test : Look at $\dfrac{\bar{X} - 240}{S/\sqrt{18}}$.

If it is more negative than $-t_{17,.05}$, reject null (that's evidence that specifications are not being met)

Otherwise, accept null.

8.25) If we take $H_0 : \mu \leq 210$
$H_1 : \mu > 210$,

then H_1 will be definitely rejected if we see $\bar{X} = 200$

If we take $H_0 : \mu \geq 210$
$H_1 : \mu < 210$,

{ ie, accept the claim unless evidence suggests o/wise }

then test is : look at $\dfrac{\bar{X} - 210}{\frac{35}{\sqrt{n}}}$,

and compare it to $-t_{n-1,.05}$; if it is more negative, reject claim.

a) $n = 25$, $\dfrac{\bar{X} - 210}{\frac{35}{\sqrt{25}}} = -1.428$

Since $t_{24,.05} = 1.711$, we do <u>not</u> reject null.

b) $n = 64$, $\dfrac{\bar{X} - 210}{\frac{35}{\sqrt{64}}} = -2.28$

Since $t_{63,.05} = +1.645$, we <u>do</u> reject null.