Math 30530, Fall 2009, Homework 2 Solutions

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- **1.** $\dfrac{30-10}{30-0} = \dfrac{2}{3}$.

- **3.** **(a)** False; in the experiment of choosing a point at random from the interval $(0, 1)$, let $A = (0, 1) - \{1/2\}$. A is not the sample space but $P(A) = 1$.
  (b) False; in the same experiment $P(\{1/2\}) = 0$ while $\{\frac{1}{2}\} \neq \emptyset$.

- **4.** $P(A \cup B) \geq P(A) = 1$, so $P(A \cup B) = 1$. This gives
$$P(AB) = P(A) + P(B) - P(A \cup B) = 1 + 1 - 1 = 1.$$

- **8.** Use induction. For $n = 1$, the theorem is trivial. Exercise 4 proves the theorem for $n = 2$. Suppose that the theorem is true for n. We show it for $n + 1$,
$$\begin{aligned} P(A_1A_2\cdots A_nA_{n+1}) &= P(A_1A_2\cdots A_n) + P(A_{n+1}) - P(A_1A_2\cdots A_n \cup A_{n+1}) \\ &= 1 + 1 - 1 = 1, \end{aligned}$$
where $P(A_1A_2\cdots A_n) = 1$ is true by the induction hypothesis, and
$$P(A_1A_2\cdots A_n \cup A_{n+1}) \geq P(A_{n+1}) = 1,$$
implies that $P(A_1A_2\cdots A_n \cup A_{n+1}) = 1$.

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- **1.** The total number of six-digit numbers is $9 \times 10 \times 10 \times 10 \times 10 \times 10 = 9 \times 10^5$ since the first digit cannot be 0. The number of six-digit numbers without the digit five is $8 \times 9 \times 9 \times 9 \times 9 \times 9 = 8 \times 9^5$. Hence there are $9 \times 10^5 - 8 \times 9^5 = 427,608$ six-digit numbers that contain the digit five.

- **3.** There are $26 \times 26 \times 26 = 17,576$ distinct sets of initials. Hence in any town with more than 17,576 inhabitants, there are at least two persons with the same initials. The answer to the question is therefore yes.

- **7.** $6/36 = 1/6$.

- **8.** **(a)** $\dfrac{4 \times 3 \times 2 \times 2}{12 \times 8 \times 8 \times 4} = \dfrac{1}{64}$. **(b)** $1 - \dfrac{8 \times 5 \times 6 \times 2}{12 \times 8 \times 8 \times 4} = \dfrac{27}{32}$.

- **13.** $(2 + 1)(3 + 1)(2 + 1) = 36$. (See the solution to Exercise 24.)

• **21.** Draw a tree diagram for the situation in which the salesperson goes from I to B first. In this situation, you will find that in 7 out of 23 cases, she will end up staying at island I. By symmetry, if she goes from I to H, D, or F first, in each of these situations in 7 out of 23 cases she will end up staying at island I. So there are $4 \times 23 = 92$ cases altogether and in $4 \times 7 = 28$ of them the salesperson will end up staying at island I. Since $28/92 = 0.3043$, the answer is 30.43%. Note that the probability that the salesperson will end up staying at island I is *not* 0.3043 because not all of the cases are equiprobable.

• **24.** Divisors of N are of the form $p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$, where $e_i = 0, 1, 2, \ldots, n_i, 1 \leq i \leq k$. Therefore, the answer is $(n_1 + 1)(n_2 + 1) \cdots (n_k + 1)$.

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• **1.** The answer is $\frac{1}{4!} = \frac{1}{24} \approx 0.0417$.

• **3.** $\frac{8!}{3!\,5!} = 56.$

• **5.** (a) $3^{12} = 531,441$. (b) $\frac{12!}{6!\,6!} = 924$. (c) $\frac{12!}{3!\,4!\,5!} = 27,720$.

• **9.** There are 8! schedule possibilities. By symmetry, in 8!/2 of them Dr. Richman's lecture precedes Dr. Chollet's and in 8!/2 ways Dr. Richman's lecture precedes Dr. Chollet's. So the answer is $8!/2 = 20,160$.

• **11.** $1 - (6!/6^6) = 0.985$.

• **15.** $\frac{m!}{(n+m)!}$.

• **20.** $\frac{2 \times 5! \times 5!}{10!} = 0.0079.$