Chapter 1

Problems

1. (a) By the generalized basic principle of counting there are

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 67{,}600{,}000$$

(b) $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 19{,}656{,}000$

4. There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are $2 \cdot 1 \cdot 2 \cdot 1 = 4$ possibilities.

5. There were $8 \cdot 2 \cdot 9 = 144$ possible codes. There were $1 \cdot 2 \cdot 9 = 18$ that started with a 4.

6. Each kitten can be identified by a code number i, j, k, l where each of i, j, k, l is any of the numbers from 1 to 7. The number i represents which wife is carrying the kitten, j then represents which of that wife's 7 sacks contain the kitten; k represents which of the 7 cats in sack j of wife i is the mother of the kitten; and l represents the number of the kitten of cat k in sack j of wife i. By the generalized principle there are thus $7 \cdot 7 \cdot 7 \cdot 7 = 2401$ kittens

8. (a) $5! = 120$
(b) $\dfrac{7!}{2!2!} = 1260$
(c) $\dfrac{11!}{4!4!2!} = 34{,}650$
(d) $\dfrac{7!}{2!2!} = 1260$

10. (a) $8! = 40{,}320$
(b) $2 \cdot 7! = 10{,}080$
(c) $5!4! = 2{,}880$
(d) $4!2^4 = 384$

11. (a) 6!
(b) 3!2!3!
(c) 3!4!

15. There are $\binom{10}{5}\binom{12}{5}$ possible choices of the 5 men and 5 women. They can then be paired up in 5! ways, since if we arbitrarily order the men then the first man can be paired with any of the 5 women, the next with any of the remaining 4, and so on. Hence, there are $5!\binom{10}{5}\binom{12}{5}$ possible results.

18. $\binom{5}{2}\binom{6}{2}\binom{4}{3} = 600$

19. (a) There are $\binom{8}{3}\binom{4}{3} + \binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees.

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

(b) There are $\binom{6}{3}\binom{6}{3} + \binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees.

(c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ possible committees. There are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves; $\binom{7}{2}\binom{5}{3}$ in which the feuding women serves; and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves.

21. $\frac{7!}{3!4!} = 35$. Each path is a linear arrangement of 4 r's and 3 u's (r for right and u for up). For instance the arrangement r, r, u, u, r, r, u specifies the path whose first 2 steps are to the right, next 2 steps are up, next 2 are to the right, and final step is up.

27. $\binom{12}{3, 4, 5} = \frac{12!}{3!4!5!}$

29. (a) (10)!/3!4!2!

(b) $3\binom{3}{2}\frac{7!}{4!2!}$

Theoretical Exercises

2. $\sum_{i=1}^{m} n_i$

8. There are $\binom{n+m}{r}$ gropus of size r. As there are $\binom{n}{i}\binom{m}{r-i}$ groups of size r that consist of i men and $r - i$ women, we see that

$$\binom{n+m}{r} = \sum_{i=0}^{r}\binom{n}{i}\binom{m}{r-i}.$$

12. Number of possible selections of a committee of size k and a chairperson is $k\binom{n}{k}$ and so $\sum_{k=1}^{n} k\binom{n}{k}$ represents the desired number. On the other hand, the chairperson can be anyone of the n persons and then each of the other $n - 1$ can either be on or off the committee. Hence, $n2^{n-1}$ also represents the desired quantity.

(i) $\binom{n}{k}k^2$

(ii) $n2^{n-1}$ since there are n possible choices for the combined chairperson and secretary and then each of the other $n - 1$ can either be on or off the committee.

(iii) $n(n - 1)2^{n-2}$

(c) From a set of n we want to choose a committee, its chairperson its secretary and its treasurer (possibly the same). The result follows since

(a) there are $n2^{n-1}$ selections in which the chair, secretary and treasurer are the same person.

(b) there are $3n(n - 1)2^{n-2}$ selection in which the chair, secretary and treasurer jobs are held by 2 people.

(c) there are $n(n - 1)(n - 2)2^{n-3}$ selections in which the chair, secretary and treasurer are all different.

(d) there are $\binom{n}{k}k^3$ selections in which the committee is of size k.

13. $(1 - 1)^n = \sum_{i=0}^{n}\binom{n}{i}(-1)^{n-1}$

18. Suppose that r labelled subsets of respective sizes $n_1, n_2, \ldots, n_r$ are to be made up from elements $1, 2, \ldots, n$ where $n = \sum_{i=1}^{r} n_i$. As $\binom{n-1}{n_1,\ldots,n_i - 1,\ldots n_r}$ represents the number of possibilities when person n is put in subset i, the result follows.