1. In how many different ways can 7 people be seated in a row if

(a) There is a group of 4 among the 7 who insist on being seated together (in any order); or

**Solution**: 4 ways to choose the four seats in a row occupied by the four who want to sit together; 4! ways to seat those four in the four selected seats; 3! ways to seat the rest, for a total of \((4)(4!)(3!) = 576\).

(b) Within the group of 4 who insist on being seated together, there are 2 who insist on being seated side-by-side.

**Solution**: 4 ways to choose the four seats in a row occupied by the four who want to sit together; 3 ways to choose the two seats in a row within the four for the two who want to sit together; 2! ways to seat those two; 2! ways to seat the other two from among the four; 3! ways to seat the rest, for a total of \((4)(3)(2!)(2!)(3!) = 288\).

2. Show, either by an algebraic or a counting argument, that

\[
k \binom{n}{k} = n \binom{n - 1}{k - 1}.
\]

**Solution**: Algebraic argument first:

\[
k \binom{n}{k} = \frac{k \cdot n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!} = \frac{(n-1)!}{(k-1)!(n-k)!} = n \binom{n - 1}{k - 1}.
\]

Now a counting argument: the left hand side counts the number of ways of selecting \(k\) people from \(n\) to form a committee, and then select one of the \(k\) to be the chair. The right hand side counts the number of ways of selecting one person from \(n\) to be chair of a committee, and then \(k - 1\) people from the remaining \(n - 1\) to join him on the committee. So both sides count the same thing — the number of committees-with-chair of size \(k\) from a group of size \(n\) — and thus are equal.