Math 30530 — Introduction to Probability

Quiz 4 – Wednesday September 21, 2011

NAME: *Solutions*

1. Show that if E and F are independent events, then $P(E^c|F) = P(E^c)$.

   Solution: $P(E^c|F) = \frac{P(E^cF)}{P(F)} = \frac{P(F)-P(EF)}{P(F)} = \frac{P(F)-P(E)P(F)}{P(F)} = \frac{P(F)(1-P(E))}{P(F)} = 1 - P(E) = P(E^c)$. It's in the third equality that we use the information that E and F are independent.

   Many were tempted to say "since we know E and F are independent, then so too are E^c and F, so $P(E^cF) = P(E^c)P(F)$". While this is true, I deducted points for it, because it essential answers the question by assuming the answer, not really in the spirit of a quiz designed to show me what you know ... (plus, although we've said that the right interpretation of independence is that no information about one event gives information about the other, we didn't actually prove all of this in class - we just showed that $P(E|F) = P(F) \Rightarrow P(E|F^c) = P(E)$; this quiz question was designed for you to show me that you understood that argument by applying it in a similar but different situation).

2. I toss a coin twice in a row. Let A be the event that the first toss results in a head, B the event that the second toss results in a head, and C the event that both tosses have the same result (either both heads or both tails). Show that A and B are independent, that A and C are independent, that B and C are independent, but that $\{A, B, C\}$ is not a set of independent events.

   Solution: $P(A) = 1/2$, $P(B) = 1/2$ and $P(C) = 1/2$; $P(AB) = 1/4$, $P(AC) = 1/4$ and $P(BC) = 1/4$. All of these (easy) facts together show that A and B, A and C and B and C are independent. To show that the triple is not an independent collection, notice that

   $$P(ABC) = 1/4 \neq 1/8 = P(ABC).$$

   Here's an intuitive way to see that the three events don't form an independent collection: if I tell you that both A and B occurred, then you know for certain that C occurred; so A and B together give information about C.