

# Math 30530 — Introduction to Probability

Quiz 6 – Wednesday November 2, 2011

NAME: \_\_\_\_\_ *Solutions* \_\_\_\_\_

I choose a random real number between 0 and 2. Let  $X$  be the number that I choose.

1. Write down the density function  $f(x)$  of  $X$ . (Be sure to specify its value for all inputs in the domain  $-\infty < x < \infty$ ).

**Solution:**

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1/2 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$$

2. Write down the cumulative distribution function  $F(x)$  of  $X$ . (Be sure to specify its value for all inputs in the domain  $-\infty < x < \infty$ .)

**Solution:** For  $x < 0$ ,  $F(x) = 0$ , and for  $x > 2$ ,  $F(x) = 1$ . For  $0 \leq x \leq 2$ ,

$$F(x) = P(X \leq x) = \int_0^x \frac{1}{2} dt = \frac{x}{2}.$$

3. For  $a$  a real number between 0 and 8, what is  $P(X^3 \leq a)$ ?

**Solution:**

$$P(X^3 \leq a) = P(X \leq a^{1/3}) = \int_0^{a^{1/3}} \frac{1}{2} dt = \frac{a^{1/3}}{2}.$$

4. Write down the density function  $g(x)$  of the random variable  $X^3$ .

**Solution:** For  $x < 0$ ,  $g(x) = 0$ , and for  $x > 8$ ,  $g(x) = 0$ . For  $0 \leq x \leq 8$ , we obtain the density function by differentiating the distribution function:

$$g(x) = \frac{d}{dx} \frac{x^{1/3}}{2} = \frac{x^{-2/3}}{6}.$$