Math 30530: Introduction to Probability, Fall 2012

Midterm Exam II

General information

When is the exam?

In class on Monday, November 19

What does the exam cover?

Ward and Gundlach:

- Chapters 17 and 18
- Chapters 23 through 26
- Chapters 28 through 32
- Chapters 35 and 36

What is the format?

There will be five or six question; all free response

What can I do to prepare?

- Review the material! Know:
  - all the things you knew for Exam I (much of this will not be explicitly examined, but will be implicitly assumed)
  - for each of the following named families of discrete random variables, know when to use a member of the family, what the parameters of the random variable are, what the possible values are, what the mass function is, and what the expectation and variance are:
    * Poisson
    * Hypergeometric
  - the assumptions underlying the Poisson process
  - density and CDF of continuous random variables, how to convert from one to the other, and how to calculate probabilities with each of them

  - joint densities of pairs of random variables, and how to calculate probabilities involving two random variables
  - independence of pairs of random variables
  - how to calculate the expectation and variance of a continuous random variable
  - how to calculate the expectation of a function of a continuous random variable, and the expectation of a function of a pair of continuous random variables
  - how to calculate the distribution function of a function of a continuous random variable, and of a pair of continuous random variables, and how to calculate the density in this case
  - the median of a continuous random variable
  - for each of the following named families of continuous random variables, know when to use a member of the family, what the parameters of the random variable are, what the possible values are, what the density function is, and what the expectation and variance are:
    * Uniform
    * Exponential
    * Normal
  - the density of the minimum of two or more independent exponential random variables
  - the difference between the standard normal and the general normal, and how to move from one to the other by a linear transformation
  - how to read probabilities off a standard normal table
  - the distribution of the sum of independent normal random variables

  As well as reviewing your class notes and the textbook (pay particular attention to the textbook's review chapters, 23 and 30), there are notes on the named continuous random variables on the course page.

- Do plenty of questions! Basically any exercise from any of the examined chapters is appropriate; in particular there are review problems at the end of Chapter 30 (what's nice is that these chapters are reviewing multiple topics, so the title of the chapter doesn't immediately give away what the questions are testing; this replicates the exam situation). By Friday, all homework solutions should be up on the course website for you to review. At the end of this document I've included some more problems (from old exams) for you to look at.

- Come talk to me! I've tentatively set office hours for the following times:
  - Wednesday 4-5, Hayes-Healy 248.
  - Thursday 10.45-12.30, Hayes-Healy 248.
  - Friday 3-3.30, in the classroom
  - Sunday 5.30-6.30, Hayes-Healy 129
  - Monday 12.35-1.45, Pasquerilla 109

Some review problems

Solutions will be posted over the weekend.

1. The time it takes for a calculus student to answer all the questions on a certain exam is an exponentially distributed random variable with mean 1 hour and 15 minutes. If 10 students are taking the exam, what is the probability that at least one of them completes it in less than 1 hour?

2. The joint density of a pair of random variables X, Y is

$$f(x,y) = \begin{cases} Cxe^{-4y} & \text{if } 0 \le x \le 4 \text{ and } 0 \le y \le \infty \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find C.

   (b) Are X and Y independent? Give a brief explanation.

   (c) Write down (but don't evaluate) an integral whose value is the probability that $X + Y \ge 4$.

   (d) Compute the mean and variance of $XY^2 + X^2Y$ (just write down the integrals that you would compute).

3. Let X and Y be independent random variables, each uniformly distributed on $[0, 1]$. Find the probability that the quadratic equation

$$t^2 - Xt + Y = 0$$

   has real roots.

4. I arrive at the bus stop by the bookstore at noon. I know that the number of hours that will pass before the bus to Midway arrives is a random variable X that is uniformly distributed on $[0, 1/2]$. The National Weather service tells me that it will begin raining sometime after noon, and that the number of hours that will pass before the rain starts is an exponentially distributed random variable Y with $\lambda = 2$.

   (a) Write down the joint density function $f(x, y)$ of X and Y.

   (b) Compute the probability that it will begin to rain before the bus arrives.

5. Let the random variables X and Y have joint density

$$f(x,y) = \begin{cases} \frac{1}{2}e^{-x}, & \text{if } x > 0 \text{ and } 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the marginal densities f_X and f_Y.

   (b) Are X and Y independent? Explain.

   (c) Find $P\{X < Y\}$.

6. A box contains a large number of nails whose lengths are normally distributed with mean 4 cm and standard deviation .1 cm. 10 nails are chosen at random from the batch. What is the probability that exactly 5 of them will be between 3.9 cm and 4.1 cm in length?

7. Dr. G. teaches on MWF 1.55-2:45 pm. On Mondays he usually makes a mistake, on average, every 10 minutes. On Wednesdays he mostly gets his act together, but still adds things incorrectly once about every 25 minutes. By Friday he gets tired and, on average, misspells a word eight times during the lecture period. On each day, the number of mistakes he makes is modeled by a Poisson random variable. If a randomly chosen lecture (equally likely to be M, W or F) happened to be surprisingly error free, what is the probability that it was delivered on Friday?

8. A random variable X has density function given by

$$f(x) = \begin{cases} \frac{3}{x^4} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Compute $E(X)$.

   (b) Compute the distribution function $F_{X^2}(t)$ of X^2. (Give its value for all t.) For $t < 1$ we have $F_{X^2}(t) = 0$. For $t \geq 1$,

   (c) Compute the density function $f_{X^2}(x)$ of X^2.